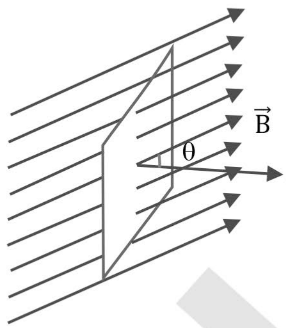
Magnetic Flux

The magnetic flux (ϕ) linked with a surface held in a magnetic field (B) is defined as the number of magnetic field lines crossing that area (A). If θ is the angle between the direction of the field and normal to the area, (area vector).

A plane of surface area $\vec{A}$ placed in a uniform magnetic field $\vec{B}$.



No. of magnetic field lines passing from any surface area is called magnetic flux.

$\phi = \vec{B} \cdot \vec{A} = BA\cos\theta$

If coil has N turns,

$\phi = NBA\cos\theta$

where, $\theta \rightarrow$ Angle between $\vec{A}$ and $\vec{B}$

Flux Linkage

If a coil has more than one turn, then the flux through the whole coil is the sum of the flux through the individual turn. If the magnetic field is uniform, the flux through one turn is $\phi = BA\cos\theta$

So, for N turns, the total flux linkage $\phi = NBA\cos\theta$

Note:

(1) Magnetic field lines are imaginary, magnetic flux is a real scalar physical quantity with dimensions

(2) Magnetic flux is a scalar quantity

(3) Units:

- (a) SI Unit : Weber (Wb)
- (b) Derived SI unit : Tesla-meter^2 (T-m^2)
- (c) CGS UNIT : Maxwell (Mx)
- (d) Conversion factor : 1 Wb = 10^8 Mx

Dimensional formula of magnetic flux

$\phi = [M L^2 T^{-2} A^{-1}]$

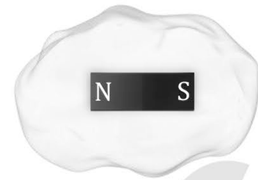
(4) If magnetic field is non-uniform than magnetic flux is given by

$\phi = \int \vec{B} \cdot d\vec{A}$

HENRY CLASSES

(5) Gauss law in magnetism: Net magnetic flux through closed surface is always zero.

$\phi = \oint \vec{B} \cdot \overrightarrow{dA} = 0$



- Since incoming field lines = Outgoing field lines
- Net flux is zero.
- Magnetic monopoles does not exist.

(6) Rate of change of flux is given by

(a) Instantaneous Rate $= \dfrac{d\phi}{dt}$ (b) Average Rate $= \dfrac{\Delta\phi}{\Delta t} = \dfrac{\phi_{Final} - \phi_{Ininal}}{\Delta t}$

Illustration 1:

A rectangular loop of area 0.06 m² is placed in a magnetic field 1.2 T with its plane inclined 30° to the field direction. Find the flux linked with plane of loop.

Solution:

Area of loop A = 0.06 m², B = 1.2 T and θ = 90° – 30° = 60°

So, the flux linked with the loop is

$\phi = BA\cos\theta = 1.2 \times 0.06 \times \cos 60° = 1.2 \times 0.06 \times 1/2 = 0.036$ Wb

Illustration 2:

A loop of wire is placed in a magnetic field $\vec{B} = 0.3\hat{j}\,T$. Find the flux through the loop if area vector is $\vec{A} = (2\hat{i} + 5\hat{j} - 3\hat{k})\ m^2$

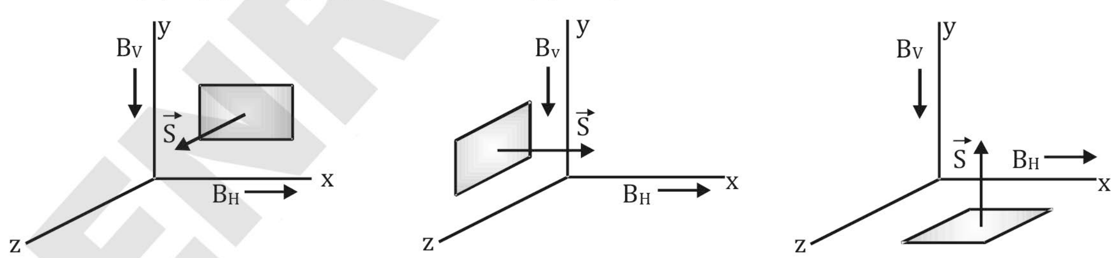
Solution:

$\vec{B} = (0\hat{i} + 0.3\hat{j} + 0\hat{k})T$ Flux linked with the surface

$\phi = \vec{B} \cdot \vec{A} = (0.3\hat{j}) \cdot (2\hat{i} + 5\hat{j} - 3\hat{k})$ T-m² = 1.5 Wb $\quad \because$ T-m² = Wb

Illustration 3:

At a given plane, horizontal and vertical components of earth's magnetic field B_H and B_V are along x and y axes respectively as shown in figure. What is the total flux of earth's magnetic field associated with an area S, if the area S is in (a) x-y plane (b) y-z plane and (c) z-x plane.



(a) (b) (c)

Solution:

$\vec{B} = \hat{i}B_H - \hat{j}B_V$ = constant, so $\quad \phi = \vec{B} \cdot \vec{S} \quad [\vec{B}$ = constant]

(a) For area in x-y plane $\vec{S} = S\hat{k}$ $\quad \phi_{xy} = (\hat{i}B_H - \hat{j}B_V).(\hat{k}S) = 0$

(b) For area S in y-z plane $\vec{S} = S\hat{i}$ $\quad \phi_{yz} = (\hat{i}B_H - \hat{j}B_V).(\hat{i}S) = B_H S$

(c) For area S in z-x plane $\vec{S} = S\hat{j}$ $\quad \phi_{zx} = (\hat{i}B_H - \hat{j}B_V).(\hat{j}S) = -B_V S$

Negative sign implies that flux is directed vertically downwards.

HENRY CLASSES

Illustration 4:

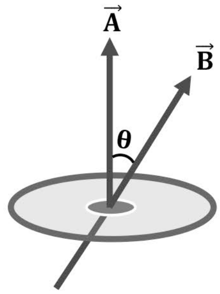
If a coil of area $\vec{A} = 3\hat{i} + 4\hat{j}$ m is placed in magnetic field $\vec{B} = 2\hat{i} + 3\hat{j} + 4k$ T. Then find the flux passing from this coil.

Solution:

$\phi_B = \vec{B} \cdot \vec{A} = (2\hat{i} + 3\hat{j} + 4k) \cdot (3\hat{i} + 4\hat{j}) = 6 + 12 = 18$ T-m^2 or 18 Wb

Illustration 5:

Circular coil of 500 turns & area 5 cm^2 is placed in uniform magnetic field B = 2T such that its area vector makes an angle of 60° with the direction of field then find flux passing from this coil?



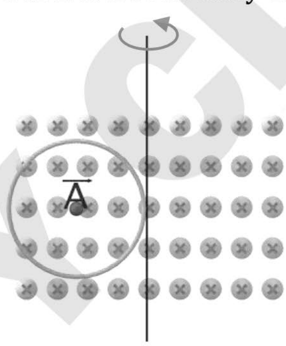
Solution:

Given: A = 5 cm^2 = 5 × 10^{-4} m^2, N = 500, B = 2T, $\theta = 60°$

$$\phi = NBA\cos\theta \Rightarrow \phi = (500)(2)(5\times10^{-4})\cos 60° \Rightarrow \phi = 0.25 \text{ Wb}$$

Illustration 6:

The change in flux through the ring of area 'A' if it is rotated by 180° in uniform magnetic field(B) as shown:

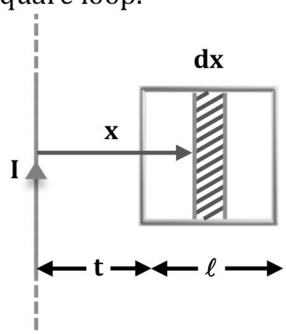


Solution:

$$\Delta\phi = \phi_2 - \phi_1 = BA\cos\theta_2 - BA\cos\theta_1 \Rightarrow BA(\cos 0° - \cos 180°) \Rightarrow 2BA$$

Illustration 7:

Find magnetic flux passing through this square loop.



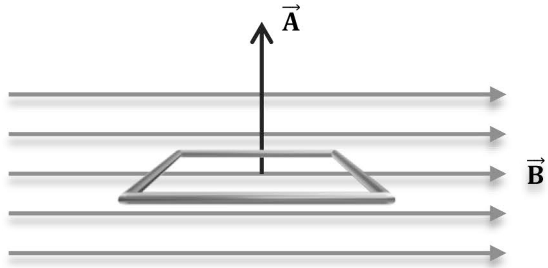
Solution:

At any distance x from wire, flux of small area is given as

$$d\phi = B(\ell dx) = \frac{\mu_0 I}{2\pi x}\ell dx \Rightarrow \phi = \int_t^{t+\ell} \frac{\mu_0 I\ell}{2\pi}\cdot\frac{dx}{x} \Rightarrow \phi = \frac{\mu_0 I\ell}{2\pi}\int_t^{t+\ell}\frac{dx}{x} \Rightarrow \phi = \frac{\mu_0 I\ell}{2\pi}\ell n\left(\frac{t+\ell}{t}\right)$$

Illustration 8:

Given coil is placed in external magnetic field then find flux through coil.



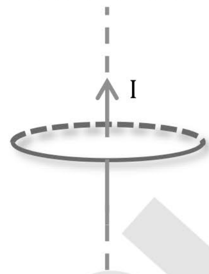
Solution:

$\phi = NBA\cos\theta$ {Here θ is angle between $\vec{B}$ & $\vec{A}$}

$\phi = NBA\cos 90° \Rightarrow \phi = 0$

Illustration 9:

Given coil is placed in external magnetic field then find flux through coil.



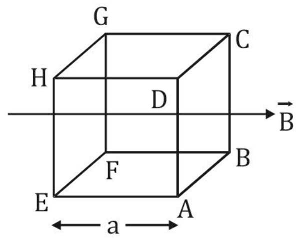
Solution:

Here angle θ between $\vec{B}$ & $\vec{A}$ is $90°$

So, flux $\phi = NBA\cos 90° = 0$

BEGINNER'S BOX-1

1. A coil of 100 turns, $5cm^2$ area is placed in external magnetic field of 0.2 Tesla (S.I.) in such a way that it makes an angle 30° with the field direction. Calculate magnetic flux through the coil (in weber).
2. A coil of N turns, A area is placed in uniform transverse magnetic field B. If it is turn through 180° about its one of the diameter in 2 seconds. Find rate of change of magnetic flux through the coil.
3. A square cube of side 'a' is placed in uniform magnetic field 'B'. Find magnetic flux through each face of the cube.



4. The magnetic field perpendicular to the plane of a loop of area 0.1 m^2 is 0.2 T. Calculate the magnetic flux through the loop.
5. The magnetic field in a certain region is given by $\vec{B} = \left(4\hat{i} - \hat{k}\right)$ tesla. How much magnetic flux passes through the loop of area $0.1m^2$ in this region if the loop lies flat in xy plane?
6. A solenoid 4cm in diameter and 20cm in length has 250 turns and carries a current of 15A. Calculate the flux through the surface of a disc of 10cm radius that is positioned perpendicular to and centered on the axis of the solenoid.

HENRY CLASSES

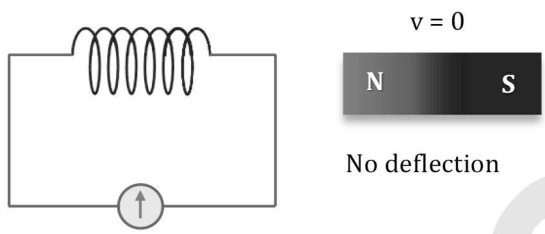
Faraday's Law of Induction and Lenz Law

Michael Faraday demonstrated the reverse effect of Oersted experiment. He explained the possibility of producing emf across the ends of a conductor when the magnetic flux linked with the conductor changes. This was termed as electromagnetic induction. The discovery of this phenomenon brought about a revolution in the field of electric power generation.

Faraday's Experiment

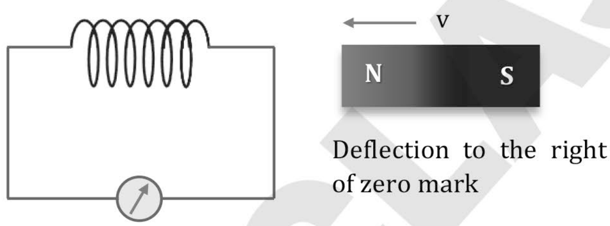
Faraday performed various experiments to discover and understand the phenomenon of electromagnetic induction. Some of them are:

- When the magnet is held stationary anywhere near or inside the coil, the galvanometer does not show any deflection.



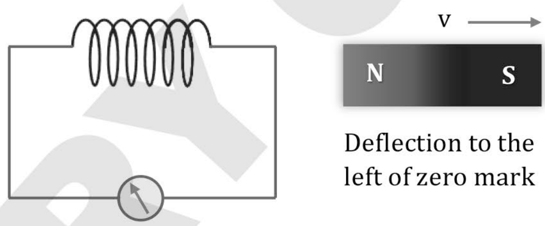
No deflection

- When the N-pole of a strong bar magnet is moved towards the coil, the galvanometer shows a deflection right to the zero mark.



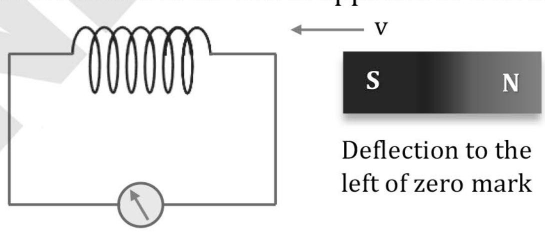
Deflection to the right of zero mark

- When the N-pole of a strong bar magnet is moved away from the coil, the galvanometer shows a deflection left to the zero mark.



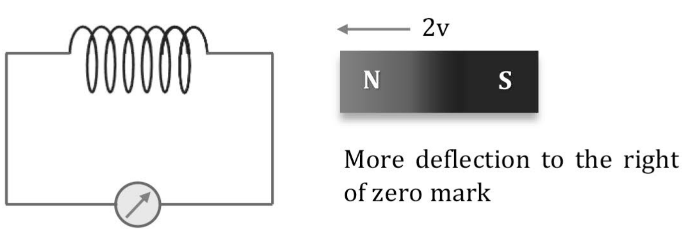
Deflection to the left of zero mark

- If the above experiments are repeated by bringing the S-pole of the, magnet towards or away from the coil, the direction of current in the coil is opposite to that obtained in the case of N-pole.



Deflection to the left of zero mark

- The deflection in galvanometer is more when the magnet moves faster and less when the magnet moves slower.



More deflection to the right of zero mark

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Conclusion

- Whenever magnetic flux changes through the coil with time an EMF get induced in it.
- Whenever there is a relative motion between the source of magnetic field (magnet) and the coil, an emf is induced in the coil. When the magnet and coil move towards each other then the flux linked with the coil increases and emf is induced. When the magnet and coil move away from each other the magnetic flux linked with the coil decreases, again an emf is induced. This emf lasts so long as the flux is changing.
- Due to this emf an electric current start to flow and the galvanometer shows deflection.
- The deflection in galvanometer last as long the relative motion between the magnet and coil continues.
- Whenever relative motion between coil and magnet takes place an induced emf produced in coil. If coil is in closed circuit then current and charge is also induced in the circuit. This phenomenon is called electromagnetic induction.

Methods of changing flux

- Changing the magnetic field through the coil
- Changing the area of the coil
- Changing area between the magnetic field and area vector.

Faraday's law

Faraday's First Law of Electromagnetic Induction states that "Whenever a conductor is placed in a varying magnetic field, an electromotive force is induced.

Faraday's Second Law of Electromagnetic Induction states that the induced emf in a coil is equal to the rate of change of flux linkage.

$|e| \propto \frac{d\phi}{dt} \quad \Rightarrow \quad |e| = k\frac{d\phi}{dt}$

where, k = 1 $\quad \Rightarrow \quad |e| = \frac{d\phi}{dt}$

Important Points

- EMF is induced in the coil irrespective of its material,

metal ring **wooden ring** **plastic ring**

If coil is conducting, current is also induced in the coil.

- If coil and source of magnetic field have relative motion than (induced emf) $\propto$ (relative velocity)

Direction of Induced Current

Lenz' Law

Direction of induced current is always in such a way that it opposes the reason by which it is produced.

$e = -\frac{d\phi}{dt}$ (–ve sign is due to Lenz Law and denotes opposition)

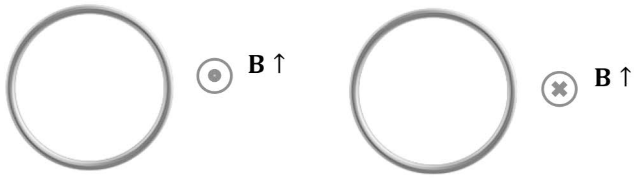
Note:

- If ϕ-t curve is given than negative of its slope will give us induced EMF.
  e = −(slope of ϕ-t curve)
- Lenz law follows conservation of energy.

HENRY CLASSES

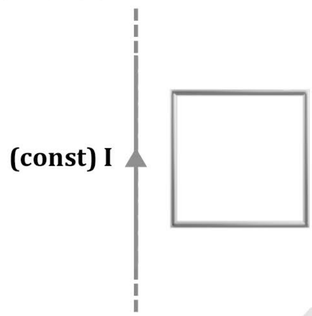
Illustration 10:

(a) Find the direction of induced current in the coil.

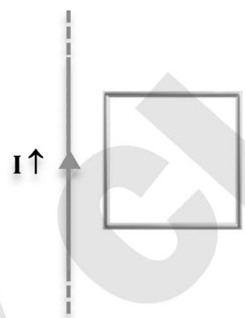


(i) (ii)

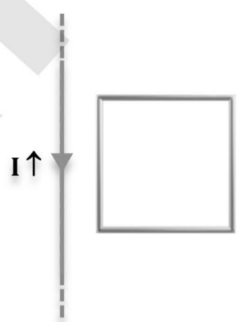
(b) Find direction of induced current in the coil.



(c) Find direction of induced current in the coil.



(d) Find direction of induced current in the coil.



Solution:

(a) B is increasing so current will induce in such a way so that it opposes of increasing B.
So, (i) induced current in the coil will be in clockwise direction.
(ii) induced current in the coil will be in Anti clockwise direction.

(b) Here magnetic field $(\vec{B})$ is constant. There will be no change in flux. So, induced current will be zero.

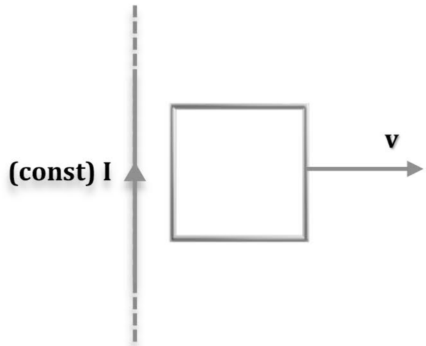
(c) Due to increase in current, magnetic field $(\vec{B})$ will be increase. So, induced current in the loop will be in Anti clockwise direction.

(d) Due to increase in current, magnetic field $(\vec{B})$ will be increase. So, induced current in the loop will be in clockwise direction.

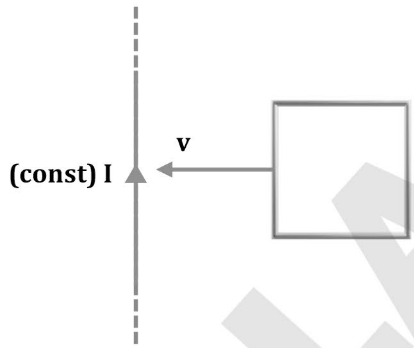
HENRY CLASSES

Illustration 11:

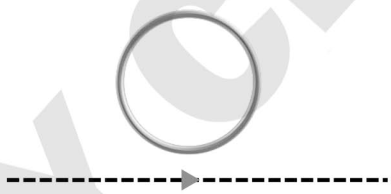
(a) Find direction of induced current in the coil.



(b) Find direction of induced current in the coil.

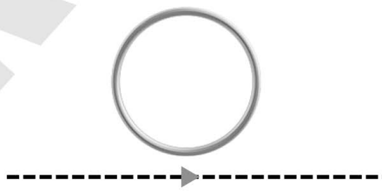


(c) Find direction of induced current in the coil.



electron beam
(constant speed)

(d) Find direction of induced current in the coil.



electron beam
(increasing speed)

Solution:

(a) Due to motion of coil magnetic field is decreasing. So current in coil will be induced in clockwise direction.

(b) Due to motion of coil magnetic field is increasing. So current in coil will be induced in Anti-clockwise direction.

(c) Speed of electron beam is constant. It means current is constant. There will be no change in flux. So induced current in loop will be zero.

(d) Speed of electron is increasing. So current will be increases and current in coil will be induced in Anti-clockwise.

HENRY CLASSES

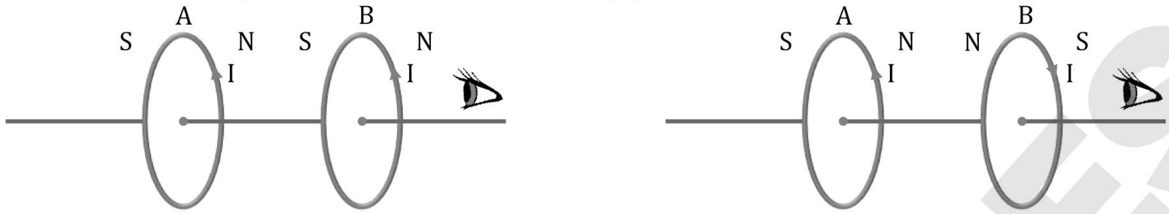
Illustration 12:

Two identical co–axial circular coils carries equal currents: –

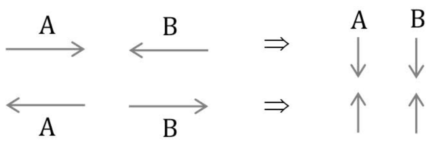
(a) In same direction (b) In opposite direction.

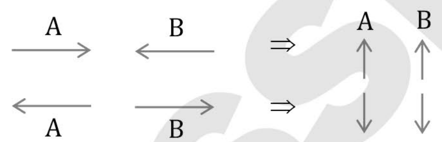
If both the coils moves towards each other and away from each other respectively then current in both coils :-

(1) Increases (2) Decreases (3) Remains same (4) None



Solution:

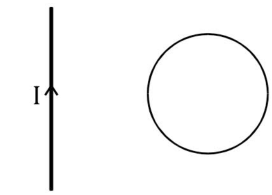




BEGINNER'S BOX-2

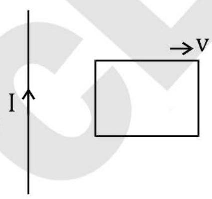
1. Find direction of induced current for the given cases :–
(Where w.r.t. = with respect to time, ob = observer =)

(i)

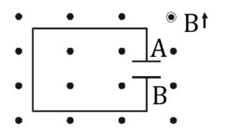


If I↑ w.r.t.

(ii)

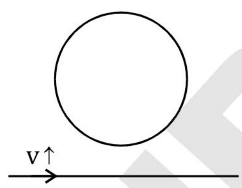


(iii)



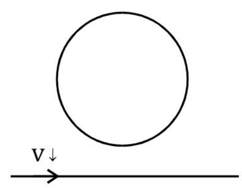
What is the nature of charge on plates of capacitor?

(iv)



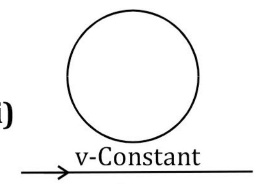
e^- bean (accelerated)

(v)



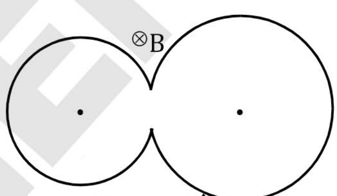
e^+ beam (retarded)

(vi)



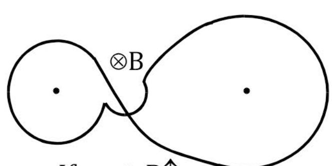
α-beam
(uniform motion)

(vii) (a)



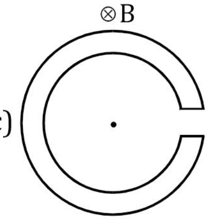
If w.r.t. B↑

(b)



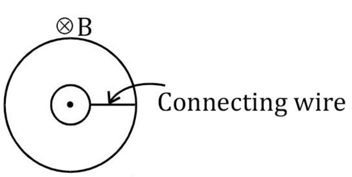
If w.r.t. B↑

(c)



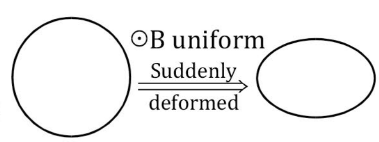
If w.r.t. B↑

(d)



If w.r.t. B↑

(viii)

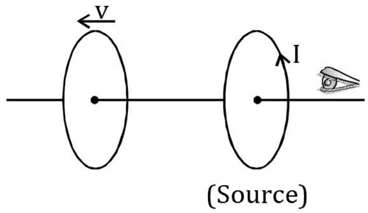


Circular loop Ellipse

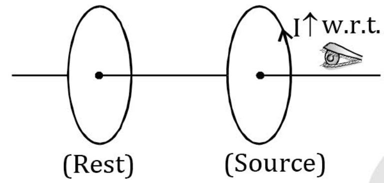
HENRY CLASSES

2. Find direction of induced current in the given cases :–

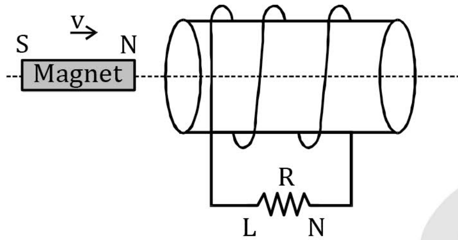
(i) (a)



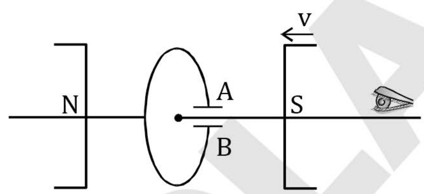
(b)



(ii) What is the direction of induced current in resistance 'R'?



(iii) What is the nature of the charge on the plates of capacitor?



Induced Parameters

(i) Induced emf (e)

(ii) Induced current (I)

(iii) Induced charge (q)

(iv) Induced heat (H)

(v) Induced electric field (E_{in})

Let for a coil its mag. flux changes by $\Delta\phi$ in time interval Δt and total resistance of coil-circuit is R.

Now rate of change of flux = $\frac{\Delta\phi}{\Delta t}$

Average induced emf $e_{av} = -\frac{\Delta\phi}{\Delta t}$

(i) Instantaneous induced emf $e = \lim_{\Delta t \to 0}\left(\frac{-\Delta\phi}{\Delta t}\right) = -\frac{d\phi}{dt}$

(ii) Induced current flow at this instant $I = \frac{e}{R}$

$$\boxed{I = \frac{-1}{R}\left(\frac{d\phi}{dt}\right)}$$

(iii) In time interval dt, induced charge $\boxed{dq = -\frac{d\phi}{R}}$

(iv) Induced heat :- $\boxed{H = \int_0^t I^2 R\,dt = \int_0^t \frac{e^2}{R}\,dt}$

HENRY CLASSES

Type of Questions on Induced E.M.F.

Induced E.M.F. $\rightarrow$ $e = -\dfrac{d\phi}{dt}$

Magnetic Flux $\rightarrow$ $\phi = NBA\cos\omega t$

Type 1: Magnetic field is changing :

$$e = -\frac{d\phi}{dt} = -NA\cos\omega t\frac{dB}{dt}$$

Type 2: Area is changing :

$$e = -\frac{d\phi}{dt} = -NB\cos\omega t\frac{dA}{dt}$$

Type 3: Radius is changing :

$$A = \pi r^2 \rightarrow \frac{dA}{dt} = 2\pi r\frac{dr}{dt}$$

$$e = -\frac{d\phi}{dt} = -NB\cos\omega t\left(2\pi r\frac{dr}{dt}\right)$$

Type 4: Angle between B and A is changing :

$$e = -\frac{d\phi}{dt} = -NAB\frac{d(\cos\omega t)}{dt}$$

BEGINNER'S BOX-3

1. Flux linked through following coils changes with respect to time then for which coil an e.m.f. is not induced:-

(1) Copper coils (2) Wood coil (3) Iron coil (4) None

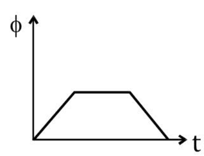
2. A coil and a magnet moves with their constant speeds 5 m/sec. and 3 m/sec. respectively, towards each other, then induced emf in coil is 16 mV. If both are moves in same direction, then induced emf in coil:-

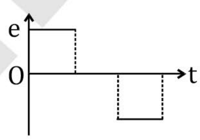
(1) 15 mV (2) 4 mV (3) 64 mV (4) Zero

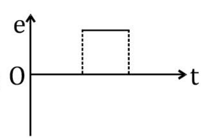
3. Magnetic flux ϕ (in Weber) linked with a closed circuit of resistance 10 ohm varies with time t (in seconds) as $\phi = 5t^2 - 4t + 1$. The induced emf in the circuit at t = 0.2 sec. is :–

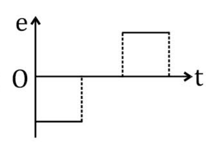
(1) 0.4 V (2) – 0.4 V (3) – 2.0 V (4) 2.0 V

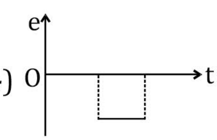
4. Magnetic flux linked through the coil changes with respect to time according to following graph, then induced emf v/s time graph for coil is :–



(1) 

(2) 

(3) 

(4) 

5. The radius of a circular coil having 50 turns is 2 cm. Its plane is normal to the magnetic field. The magnetic field changes from 2T to 4T in 3.14 sec. The induced emf in coil will be :–

(1) 0.4V (2) 0.04V (3) 4 mV (4) 0.12 V

HENRY CLASSES

6. Magnetic field changes at the rate of 0.4 T/sec. in a square coil of side 4 cm. kept perpendicular to the field. If the resistance of the coil is $2 \times 10^{-3}\ \Omega$, then induced current in coil is :–
(1) 0.16 A (2) 0.32 A (3) 3.2 A (4) 1.6 A

7. A short bar magnet allowed to fall along the axis of horizontal metallic ring. Starting from rest, the distance fallen by the magnet in one second may be :–
(1) 4.0 m. (2) 5.0 m. (3) 6.0 m. (4) 7.0 m.

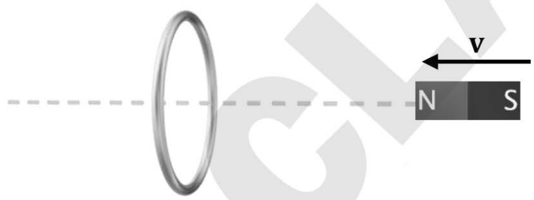
8. In a circuit a coil of resistance 2Ω, then magnetic flux changes from 2.0Wb to 10.0Wb in 0.2 sec. The charge flow in the coil during this time is :–
(1) 5.0 C (2) 4.0 C (3) 1.0 C (4) 0.8 C

9. A circular loop of radius 2cm, is placed in a time varying magnetic field with rate of 2T/sec. Then induced electric field in this loop will be :-
(1) 0 (2) 0.02 V/m (3) .01 V/m (4) 2 V/m

Lenz Law and Conservation of Energy

1. When the North Pole of the Bar magnet comes towards the coil, it experience a repulsive force due to which its speed will decrease.
2. To move the magnet towards the coil with constant speed, some part of mechanical work has to be done to overcome the force of repulsion.
3. This mechanical work is converted into electrical energy.
4. This electrical energy is converted into heat energy due to Joule's Effect.

Mathematical Analysis



Here Kinetic Energy gets converted into Electrical energy which further gets converted into Thermal energy

$$\frac{1}{2}Mv^2 = i^2R\Delta t = ms\Delta\theta$$

Illustration 13:
A bar magnet is moving towards a circular coil with a kinetic energy of 1180 J. If mass of the silver ring is 1 Kg and its specific heat is 236 J/kg °C than find rise in temperature of the ring.

Solution:
$K.E = ms\Delta\theta \Rightarrow 1180 = (1)(236)\Delta\theta \Rightarrow \Delta\theta = 5$ So, temperature rise will be 5°C.

Illustration 14:
A bar magnet of mass m is given initial speed v_0 towards the ring as shown, if repulsive force acting on it is given by F = –bv, then find its speed after time t.

Solution:
Suppose its speed after time t is v'

$F = -bv \Rightarrow m\frac{dv}{dt} = -bv \quad (F = ma = m\frac{dv}{dt})$

$\frac{dv}{v} = -\frac{b}{m}dt \Rightarrow \int_{v_0}^{v'}\frac{dv}{v} = -\frac{b}{m}\int_0^t dt$

$\ell n\frac{v'}{v_0} = -\frac{bt}{m} \Rightarrow v' = v_0 e^{-\frac{b}{m}t}$

So, speed of bar magnet after time t will be $v' = v_0 e^{-\frac{b}{m}t}$.

HENRY CLASSES

Types of EMI

For a loop flux, (ϕ = BA cosθ) changes w.r.t. time in following three manner and according to it electromagnetic induction is classified in three ways :-

(i) If (A, θ) $\rightarrow$ const & $\frac{dB}{dt} \rightarrow \frac{d\phi}{dt} \Rightarrow$ **Static EMI** → (1) Self Induction (In this case EMI occurs for rest coil); → (2) Mutual Induction

(ii) If (B, θ) $\rightarrow$ const & $\frac{dA}{dt} \rightarrow \frac{d\phi}{dt} \Rightarrow$ **Dynamic EMI** (In this case EMI occurs for a moving straight wire)

(iii) If (A, B) $\rightarrow$ const & $\frac{d\theta}{dt} \rightarrow \frac{d\phi}{dt} \Rightarrow$ **Periodic EMI** (In this case E.M.I. occurs for a rotating coil)

Static E.M.I. $\Rightarrow \frac{dI}{dt} \rightarrow \frac{dB}{dt} \rightarrow \frac{d\phi}{dt} \Rightarrow$ Static EMI

Self Induction

When current through the coil changes, with respect to time then magnetic flux linked with the coil also changes with respect to time. Due to this an emf and a current induced in the coil. According to Lenz law induced current opposes the change in magnetic flux. This phenomenon is called self-induction and a factor by virtue of which the coil shows opposition for change in magnetic flux called self-inductance of coil. Considering this coil circuit in two cases :

Case–I : Current through the coil is constant :–

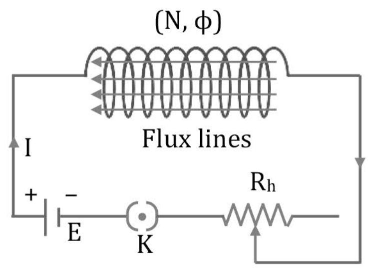
If I $\rightarrow$ B $\rightarrow \phi \rightarrow$ Const. $\Rightarrow$ No EMI

total flux of coil (Nϕ) $\propto$ current through the coil

$N\phi \propto I$

$N\phi = LI$

$$\boxed{L = \frac{N\phi}{I} = \frac{NBA}{I} = \frac{\phi_{Total}}{I}}$$, Where L : self inductance of coil



Important Points

- Self inductance is scalar quantity.
- Its S.I. Unit is henry (H) or Wb/A. Dimensions : $[M^1 L^2 T^{-2} A^{-2}]$
- Due to self-induction, a coil opposes change in its current (due to induced emf), hence, It is also called Inertia of Electricity.
- It is analogous to mass in mechanics.
- L does not depend on
  (a) Flux (ϕ) (b) Current (I)
- L depends on
  (a) Geometry of inductor (b) Medium ($\mu_0 = \mu_0\mu_r$)

Self inductance of solenoid

Let the volume of the solenoid be V, the number of turns per unit length be n. Let a current i be flowing in the solenoid.

Magnetic field in the solenoid is given as $B = \mu_0 ni$.

The magnetic flux through one turn of solenoid $\phi = \mu_0 niA$

The total magnetic flux through the solenoid = $N\phi = N\mu_0 niA = \mu_0 n^2 i A \ell$

Self inductance, $L = \frac{\phi}{i} = \mu_0 n^2 A\ell = \mu_0 n^2 V$

Inductance per unit volume = $\mu_0 n^2$.

A : Cross sectional area
N : number of turns
l : length of solenoid
n : turn density

HENRY CLASSES

Case–II : Induced EMF in self induction

If current through the coil changes w.r.t. time

$$\frac{dI}{dt} \to \frac{dB}{dt} \to \frac{d\phi}{dt}$$

$$N\phi = LI$$

$-N\frac{d\phi}{dt} = -L\frac{dI}{dt}$, where $-N\frac{d\phi}{dt}$ called self induced emf of coil 'e_S'

$$\boxed{e_s = -L\frac{dI}{dt}}$$

Note: Self inductance is the physical property of the loop due to which it opposes the change in current that means it tries to keep the current constant. Current cannot change suddenly in the inductor.

Illustration 15:

The current in a solenoid of 240 turns, having a length of 12 cm and a radius of 2 cm, changes at the rate of 0.8 As^{-1}. Find the emf induced in it.

Solution:

$$|E| = L\frac{dI}{dt} = \frac{\mu_0 N^2 A}{\ell}\cdot\frac{dI}{dt} = \frac{4\pi\times10^{-7}\times(240)^2\times\pi\times(0.02)^2}{0.12}\times0.8 = 6\times10^{-4}\text{ V}$$

BEGINNER'S BOX-4

1. The value of self inductance of a coil is 5H. The value of current changes from 1A to 2A in 5 sec., then value of induced emf in it :–

(1) 10V (2) 0.1V (3) 1.0V (4) 100V

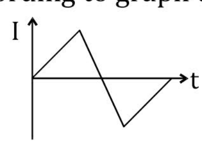
2. A coil of self inductance 2H carries a 2A current. If direction of current is reversed in 1 sec., then induced emf in it :–

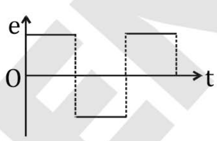
(1) – 8V (2) 8V (3) – 4V (4) Zero

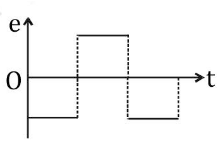
3. For a coil having L = 2mH, current flow through it is $I = t^2e^{-t}$ then the time at which emf becomes zero:–

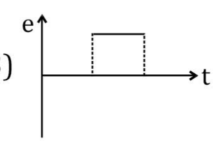
(1) 2 sec. (2) 1 sec. (3) 4 sec. (4) 3 sec.

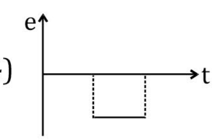
4. Current through the coil varies according to graph then induced emf v/s time graph is



(1) 

(2) 

(3) 

(4) 

5. A solenoid have the self inductance 2H. If length of the solenoid is doubled having turn density and area constant then new self inductance is :–

(1) 4H (2) 1H (3) 8H (4) 0.5 H

6. A solenoid wound over a rectangular frame. If all the linear dimensions of the frame are increased by a factor 3 and the number of turns per unit length remains the same, the self inductance increased by a factor of :–

(1) 3 (2) 9 (3) 27 (4) 63

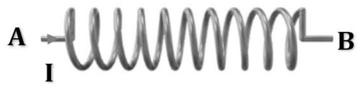
7. A coil of inductance 2 H has a current of 5.8 A. The flux in weber through the coil is :-

(1) 0.29 (2) 2.9 (3) 3.12 (4) 11.6

HENRY CLASSES

Inductor

1. A circuit element having a fixed value of inductance is known as inductor
2. It is represented by 
3. Function of inductor is to oppose the change in current in the circuit
4. Potential difference across an inductor in the direction of current is $= -L\frac{di}{dt}$



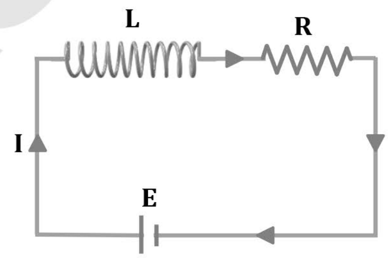
(a) If $\frac{di}{dt}$ is +ve, potential drops from A to B.

(b) If $\frac{di}{dt}$ is –ve, potential drops from B to A.

(c) If $\frac{di}{dt}$ is 0, potential between A and B is zero.

Power in an Inductor

Battery that establishes current in an inductor, works against back emf.
Part of energy supplied by the battery is stored in the inductor.
Let's apply KVL,



$$E - L\frac{dI}{dt} - IR = 0$$

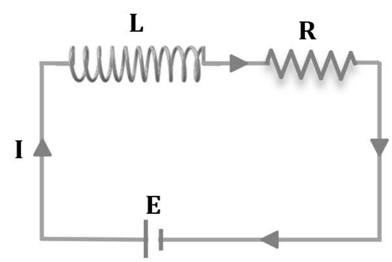
$$E = L\frac{dI}{dt} + IR$$

Instantaneous power supplied by battery,

$P = VI = EI$ (Power dissipated in the resistor = I^2R)

$P = LI\frac{dI}{dt} + I^2R$ (Power supplied to the inductor = $LI\frac{dI}{dt}$)

Energy Stored in an Inductor



$LI\frac{dI}{dt}$ (Power supplied to the inductor)

$$\frac{dU}{dt} = LI\frac{dI}{dt}$$

$$dU = LI\,dI$$

$$\int dU = \int LI\,dI$$

$$U = \frac{1}{2}LI^2$$

Energy Density of Inductor

Energy stored in the solenoid is:

$$U = \frac{1}{2}LI^2$$

Where, $L = \mu_0 n^2 V$ & $B = \mu_0 nI$ {Here V is volume of Inductor}

$$U = \frac{1}{2}(\mu_0 n^2 V)\left(\frac{B}{\mu_0 n}\right)^2 \Rightarrow \frac{U}{V} = \frac{B^2}{2\mu_0}$$

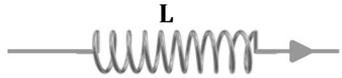
HENRY CLASSES

Important Points

- Above result is derived for an inductor but is true in general for any system.
- Magnetic energy density in free space $\frac{U}{V} = \frac{B^2}{2\mu_0}$

  Electric Energy density in free space $\frac{U}{V} = \frac{1}{2}E^2\varepsilon_0$

Inductor Behavior as a Battery



Case 1: When current is increasing

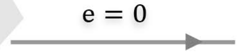
$$e = -L\frac{dI}{dt}$$

Case 2: When current is decreasing

$$e = -L\frac{dI}{dt}$$

Case 3: When current is constant

$$e = -L\frac{dI}{dt}$$



Note: If there is a resistance in the inductor (resistance of the coil of inductor) then :

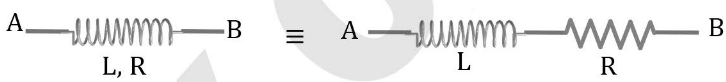
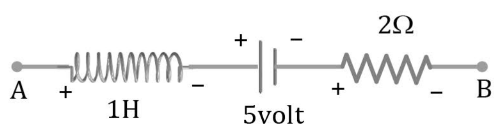


Illustration 16:

A B is a part of circuit. Find the potential difference $V_A - V_B$ if



(i) current i = 2A and is constant

(ii) current i = 2A and is increasing at the rate of 1 amp/sec.

(iii) current i = 2A and is decreasing at the rate 1 amp/sec.

Solution:

$$L\frac{di}{dt} = 1\frac{di}{dt}$$

writing KVL from A to B

$$V_A - 1\frac{di}{dt} - 5 - 2i = V_B$$

(i) Put $i = 2, \frac{di}{dt} = 0$

$V_A - 5 - 4 = V_B$ $\therefore$ $V_A - V_B = 9\text{volt}$

(ii) Put $i = 2, \frac{di}{dt} = 1; V_A - 1 - 5 - 4 = V_B$ or $V_A - V_B = 10V_0$

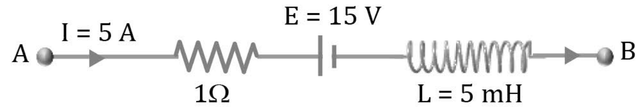
(iii) Put $i = 2, \frac{di}{dt} = -1; V_A + 1 - 5 - 2\times 2 = V_B$ or $V_A = 8\text{Volt}$

HENRY CLASSES

Illustration 17:

Find V_A-V_B in the given circuit if-

Current is decreasing at the rate 10^3 A/s.



Solution:

Given:

$$\frac{di}{dt} = -10^3$$

Written KVL from A to B

$$V_A - iR - E - L\frac{di}{dt} = V_B$$

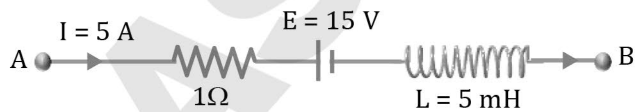
$$V_A - (5)(1) - 15 - (5\times10^{-3})(-10^{-3}) = V_B$$

$$V_A - V_B = 15 \text{ Volt}$$

Illustration 18:

Find V_A-V_B in the given circuit if-

Current is increasing at the rate 10^3 A/s.



Solution:

Given:

$$\frac{di}{dt} = 10^3$$

Written KVL from A to B

$$V_A - iR - E - L\frac{di}{dt} = V_B$$

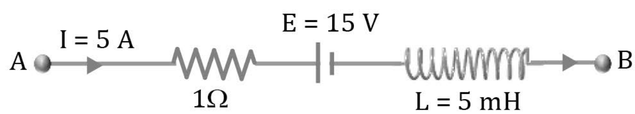
$$V_A - (5)(1) - 15 - (5\times10^{-3})(10^3) = V_B$$

$$V_A - V_B = 25 \text{ Volt}$$

Illustration 19:

Find V_A-V_B in the given circuit if-

Current is constant.



Solution:

Given:

$$\frac{di}{dt} = 0$$

Written KVL from A to B

$$V_A - iR - E - L\frac{di}{dt} = V_B$$

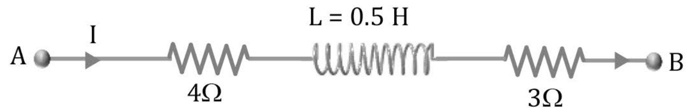
$$V_A - (5)(1) - 15 - (5\times10^{-3})(0) = V_B$$

$$V_A - V_B = 20 \text{ Volt}$$

HENRY CLASSES

Illustration 20:

Find V_A-V_B in the given circuit, at $t = 2\ s$, if $I = (t^2 + 2)\ A$.



Solution:

Given:

t = 2, $I = (t^2 + 2)$

$$\frac{dI}{dt} = 2t$$

$$\frac{dI}{dt}\ (\text{at } t = 2) = 2\ (2) = 4$$

Written KVL from A to B

$$V_A - IR - L\frac{di}{dt} - IR = V_B$$

$$V_A - (t^2+2)(4) - (0.5)(2t) - (t^2+2)(3) = V_B$$

Put t = 2 (given)

$$V_A - (2^2+2)(4) - (0.5)(2\times 2) - (2^2+2)(3) = V_B$$

$$V_A - V_B = 44 \text{ Volt}$$

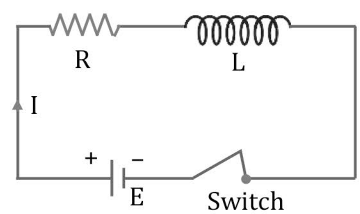
L-R Circuit Analysis

Case I : Current Growth :-

Consider an inductance L and a resistance R (including the resistance of the coil L) connected in series to a battery of emf E. When the switch S is closed, the current in the circuit begins to grow. After the key is closed the current changes from zero to some value. The current rises gradually rather than instantly. It takes some time before the current reaches its steady value $I_0 = E/R$. The effect of the inductance in a dc circuit is to increase the time taken by the current to reach its limiting value I_0.

At any instant, Kirchoff's voltage law for the loop gives

$$E - L\frac{dI}{dt} = RI$$



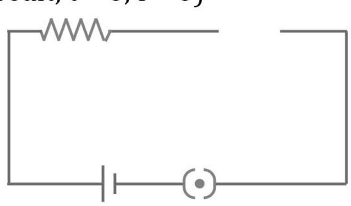
On rearranging the equation, we get

$$\frac{dI}{\frac{E}{R} - I} = \frac{R}{L}dt$$

On integrating both the sides we get

$$\boxed{I = I_0(1 - e^{-t/\lambda})} \qquad \text{where } I_0 = \frac{E}{R} \text{ and } \lambda = \frac{L}{R}$$

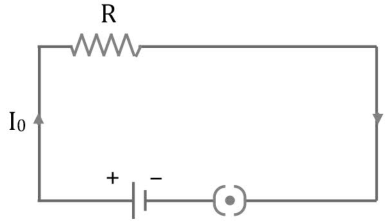
(i) Just after the closing of the key inductance behaves like open circuit and current in circuit is zero. (Open circuit, t = 0, I = 0)



(Inductor provide infinite resistance)

HENRY CLASSES

(ii) Some time after closing of the key inductance behaves like simple connecting wire (short circuit) and current in circuit is constant.



(Short circuit, $t \to \infty$, $I \to I_0$)

(Inductor provide zero resistance)

$$\boxed{I_0 = \frac{E}{R}}$$ (Final, steady, maximum or peak value of current)

Sp. Note : Peak value of current in circuit does not depends on self inductance of coil.

(iii) **Time constant of circuit (l) :** $\boxed{\lambda = \frac{L}{R}}$ Its SI unit is second(s)

It is a time in which current increases up to 63% or 0.63 times of peak current value.

(iv) **Half life (T) :** It is a time in which current increases upto 50% or 0.50 times of peak current value.

$$I = I_0(1 - e^{-T/\lambda})$$

$t = T,\ I = \frac{I_0}{2}$ $\qquad \frac{I_0}{2} = I_0(1 - e^{-T/\lambda})$

$\Rightarrow \quad e^{-T/\lambda} = \frac{1}{2}$

$\Rightarrow \quad e^{T/\lambda} = 2$

$$\frac{T}{\lambda}\log_e e = \log_e 2$$

$$\boxed{\begin{array}{l} T = 0.693\lambda \\ T = 0.693\frac{L}{R}\ \text{sec} \end{array}}$$

(v) **Rate of growth of current at any instant :–**

$$\boxed{\left(\frac{dI}{dt}\right) = \frac{E}{L}(e^{-t/\lambda})}$$

$$\boxed{t = 0 \quad \Rightarrow \quad \left(\frac{dI}{dt}\right)_{max} = \frac{E}{L}}$$

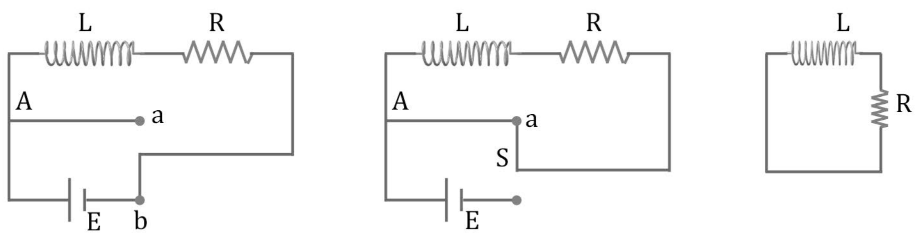
$$\boxed{t \to \infty \quad \Rightarrow \quad \left(\frac{dI}{dt}\right)_{min} \to 0}$$

Sp. Note : Maximum or initial value of rate of growth of current does not depends upon resistance of coil.

Case II : Current Decay

Consider the arrangement shown in figure (A). The sliding switch S can be slided up and down. Let the switch S connect the point b. The circuit is complete and a steady current $i = I_0$ is maintained through the circuit. Suddenly at t = 0, the switch S is moved to connect the point a. This completes the circuit through the wire Aa and disconnects the battery from the circuit [Figure (B)]. The special arrangement of the switch ensures that the circuit through the wire Aa is completed before the battery is disconnected. (Such a switch is called make before break switch). The equivalent circuit is redrawn in figure (C).

HENRY CLASSES



(A) **(B)** **(C)**

As the battery is disconnected, the current decreased in the circuit. This induced as emf is the inductor. As this is only emf in the circuit, we have

$$-L\frac{dI}{dt} = RI \qquad \text{or} \qquad \frac{dI}{I} = -\frac{R}{L}dt$$

on integrating both the sides, we get

$$\boxed{I = I_0 e^{\frac{-Rt}{L}} = I_0 e^{-t/\lambda}}$$

where λ = L/R is the time constant of the circuit.

(Just after opening of key) $t = 0 \Rightarrow I = I_0 = \frac{E}{R}$

(Some time after opening of key) $t \to \infty \Rightarrow I \to 0$

(i) **Time constant (λ) :–** It is a time in which current decreases up to 37% or 0.37 times of peak current value.

$$\boxed{\lambda = \frac{L}{R}\sec}$$

(ii) **Half life (T) :–** It is a time in which current decreases upto 50% or 0.50 times of peak current value.

$$\boxed{T = (0.693)\lambda \sec}$$

(iii) **Rate of decay of current at any instant :–**

$$\boxed{\left(-\frac{dI}{dt}\right) = \frac{E}{L}(e^{-t/\lambda})}$$

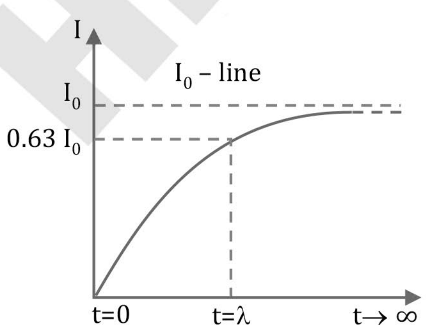
→ $t = 0 \Rightarrow \left(-\frac{dI}{dt}\right)_{max} = \frac{E}{L}$

→ $t \to \infty \Rightarrow \left(-\frac{dI}{dt}\right)_{min} \to 0$

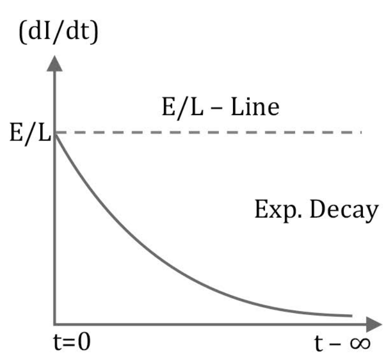
Special graph for R–L circuit :–

Current Growth :–

(a)



(b)



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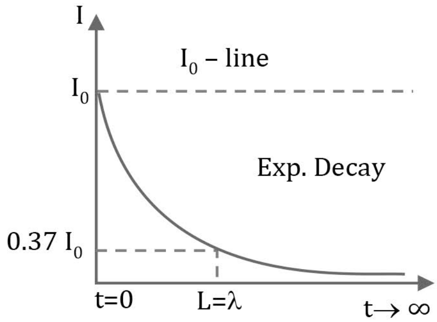
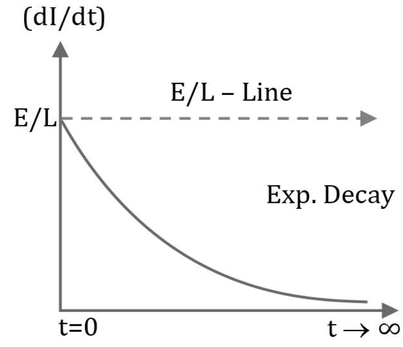
Current decay :–

(a)



(b)

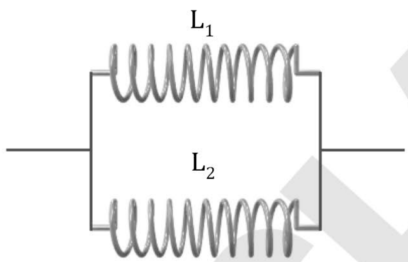


Combination of Inductors

Series Combination

$L_{eq} = L_2 + L_2$

Parallel Combination

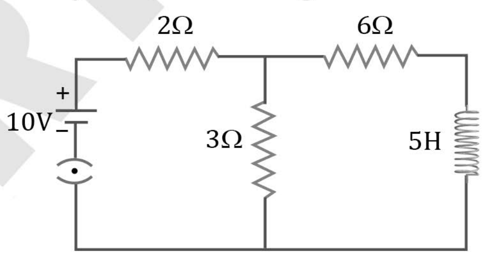


$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

Note: If an inductor is cut into 2 parts, its time constant remains same.

Illustration 21:

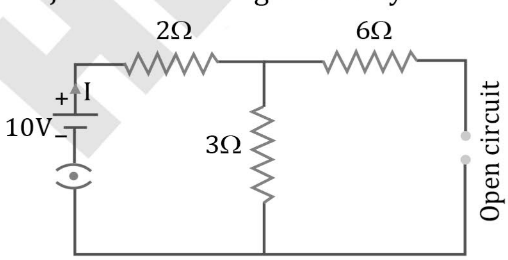
Calculate current, which given by battery for the following circuit.



(a) Just after closing of the key. (b) Some time after closing of the key

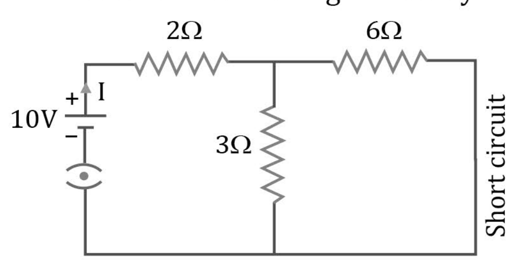
Solution:

(a) Just after closing of the key:-



Current $I = \frac{E}{r_{net}} = \frac{10}{2+3} = 2\,A$

(b) Sometime after Closing of the key:-

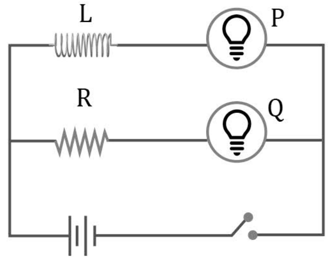


Current $I' = \frac{E}{r_{net}} = \frac{10}{4} = 2.5\,A$, Where $r_{net} = 2 + \frac{3 \times 6}{3+6}$

HENRY CLASSES

Illustration 22:

Figure shows an inductor L, a resistor R connected in parallel to a battery through a switch. The resistance of resistor R is same as that of the coil that makes L. Two identical bulb are put in each arm of the circuit.



(a) Which of two bulbs lights up earlier when S is closed?

(b) Will the bulbs be equally bright after some time?

Solution:

(i) When switch is closed induced e.m.f. in inductor i.e. back e.m.f. delays the glowing of bulb P so bulb Q light up earlier.

(ii) Yes. At steady state inductive effect becomes meaningless so both identical bulbs become equally bright after some time.

BEGINNER'S BOX-5

1. L, C and R respectively indicate inductance, capacitance and resistance. Select the combination, which does not have dimensions of frequency :-

(1) 1/RC (2) R/L (3) $1/\sqrt{LC}$ (4) C/L

2. A coil of 10 H inductance and 5 Ω resistance is connected to 5 volt battery in series. The current in ampere in circuit 2 seconds after switched is on :-

(1) e^{-1} (2) $(1-e^{-1})$ (3) (1–e) (4) e

3. An L-R circuit consists of an inductance of 8mH and a resistance of 4Ω. The time constant of the circuit is:-

(1) 2ms (2) 12ms (3) 32ms (4) 500 s

4. In an L -R circuit, time constant is that time in which current grows from zero to the value (Where I_o is the steady state current) :-

(1) 0.63 I_o (2) 0.50 I_o (3) 0.37I_o (4) I_o

5. An inductor of 20 H and a resistance of 10 Ω, are connected to a battery of 5 volt in series, then initial rate of change of current is :-

(1) 0.5 amp/s (2) 2.0 amp/s (3) 2.5 amp/s (4) 0.25 amp/s

6. A coil of L=5x10^{-3} H and R=18 Ω is abruptly supplied a potential of 5 volts. What will be the rate of change of current in 0.001 second? ($e^{-3.6}$ = 0.0273)

(1) 27.3 amp/sec. (2) 27.8 amp/sec. (3) 2.73 amp/sec. (4) 2.78 amp/sec.

7. A coil of inductance 8.4 mH and resistance 6Ω is connected to a 12V battery in series. The current in the coil is 1.0A at approximately the time :-

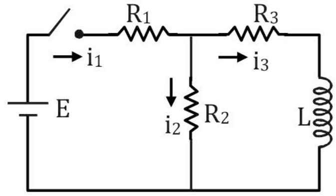
(1) 500s (2) 20s (3) 35ms (4) 1ms

HENRY CLASSES

8. The dimensions of combination $\frac{L}{CVR}$ are same as dimensions of :-

(1) Change (2) Current (3) Charge^{-1} (4) Current^{-1}

9. In the circuit shown in adjoining fig E = 10V, $R_1 = 1\Omega$ $R_2 = 2\Omega$, $R_3 = 3\Omega$ and L = 2H. Calculate the value of current i_1, i_2 and i_3 immediately after key S is closed:-



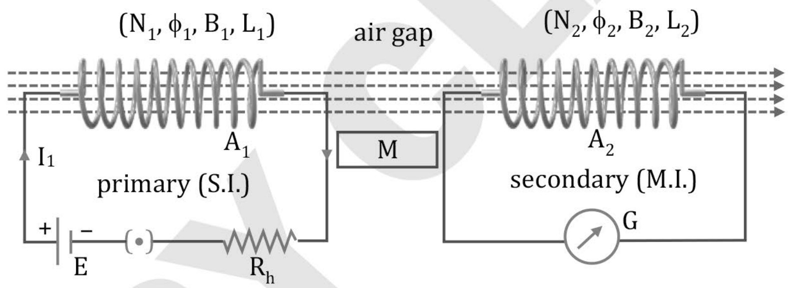
(1) 3.3 amp, 3.3 amp, 3.3 amp (2) 3.3 amp, 3.3 amp, 0 amp

(3) 3.3 amp, 0 amp, 0 amp (4) 3.3 amp, 3.3 amp 1.1 amp

Mutual Induction

Basic Concept :

Whenever current passing through primary coil or circuit change with respect to time then magnetic flux in neighbouring secondary coil or circuit will also changes with respect to time. According to Lenz Law for opposition of flux change an emf and a current induced in the neighbouring coil or circuit. This phenomenon called as 'Mutual induction'.



Due to Air gap, $\phi_2 < \phi_1$ always and $\phi_2 < B_1A_2$ $(\theta = 0^\circ)$.

Case–I : When current through primary is constant :–

Total flux of secondary is directly proportional to current flow through the primary coil

$$N_2\phi_2 \propto I_1$$

$$N_2\phi_2 = MI_1$$

$$M = \frac{N_2\phi_2}{I_1} = \frac{N_2B_1A_2}{I_1} = \frac{(\phi_T)_s}{I_p}$$, Where M : mutual inductance of circuits.

The units and dimension of M are same as 'L'.

Important Points

- Scalar Quantity.
- SI Unit is Henry (H)
- Mutual inductance of two coil does not depend on current through primary or flux through secondary coil.
- Mutual inductance is combined property of primary and secondary coil and is same for both coil.

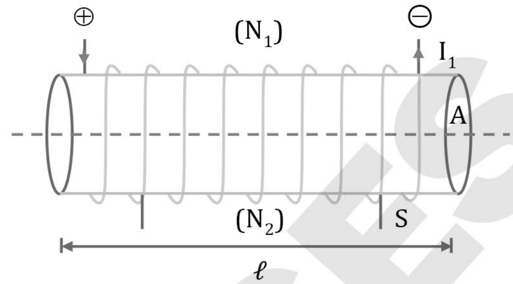
HENRY CLASSES

Different mutual inductances :–

(a) In terms of their number of turns (b) In terms of their self inductances

(a) In terms of their number of turns (N_1 , N_2) :-

(1) Two co–axial solenoids $\left(M_{S_1S_2}\right)$:-

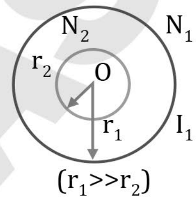


$$M_{S_1S_2} = \frac{N_2B_1A}{I_1} = \frac{N_2}{I_1}\left(\frac{\mu_0N_1I_1}{\ell}\right)A$$

where $B_1 = \frac{\mu_0N_1I_1}{\ell}$

$$\Rightarrow \quad M_{S_1S_2} = \left(\frac{\mu_0N_1N_2A}{\ell}\right)$$

(2) Two concentric and coplanar coils $\left(M_{C_1C_2}\right)$:-



$$M_{C_1C_2} = \frac{N_2B_1A_2}{I_1}$$

where $B_1 = \frac{\mu_0N_1I_1}{2r_1}$ & $A_2 = \pi r_2^2$

$$M_{C_1C_2} = \frac{N_2}{I_1}\left(\frac{\mu_0N_1I_1}{2r_1}\right)(\pi r_2^2)$$

$$\Rightarrow \quad M_{C_1C_2} = \left(\frac{\mu_0N_1N_2\pi r_2^2}{2r_1}\right)$$

(b) In terms of their self inductances (L_1, L_2) :-

For two magnetically coupled coils :-

$M = K\sqrt{L_1L_2}$, where 'K' is coupling factor between two coils and its range $0 \le K \le 1$

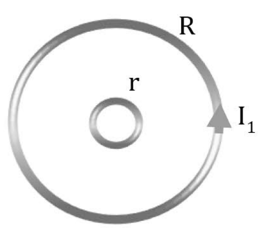
For ideal coupling $K_{max} = 1$ $\Rightarrow$ $M_{max} = \sqrt{L_1L_2}$ (Where M is geometrical mean of L_1 & L_2)

For real coupling ($0 < K < 1$) $\Rightarrow$ $M = K\sqrt{L_1L_2}$

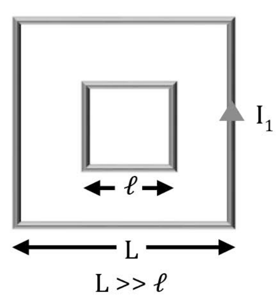
Value of coupling factor 'K' decides from fashion of coupling (how flux linked with another).

Illustration 23:

Find mutual inductance of following systems-



(a)



(b)

HENRY CLASSES

Solution:

(a)
$$B_1 = \mu_0 i_1$$
$$\phi_1 = B_1 A_2$$
$$\phi_1 = \left(\frac{\mu_0 I}{2R}\cdot \pi r^2\right)$$
$$M = \frac{\phi_1}{i_1} = \frac{\mu_0 \pi r^2}{2R}$$

(b)
$$B_1 = \sqrt{2}\,\frac{\mu_0 i_1}{\pi L}$$
$$\phi_2 = B_1 A_2$$
$$\phi_2 = \left(\sqrt{2}\,\frac{\mu_0 i_1}{\pi L}\right)\left(\ell^2\right)$$
$$M = \frac{\phi_2}{i_1} = \sqrt{2}\,\frac{\mu_0 \ell^2}{\pi L}$$

Coupling Factor

The mutual induction also depends on the relative orientation of coils.

Coupling factor represents the fraction of flux associated with secondary coil due to primary coil.

$$K = \frac{\phi_S}{\phi_P} \le 1$$

Relation between L and M,

$$M = K\sqrt{L_1 L_2}$$

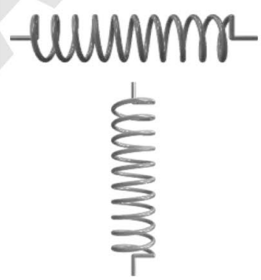
$K = \frac{\phi_S}{\phi_P} = 1$	$K = \frac{\phi_S}{\phi_P} < 1$	$K = \frac{\phi_S}{\phi_P} = 0$
		

Illustration 24:

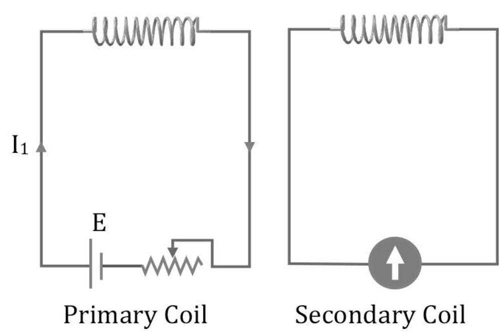
Self inductance of two coils are 2H & 8H. If 50% flux of primary coil is linked with secondary coil, then find the coefficient of mutual inductance.

Solution:

$$M = K\sqrt{L_1 L_2} \quad \Rightarrow \quad M = \left(\frac{1}{2}\right)\sqrt{(2)(8)} \quad \Rightarrow \quad M = 2H$$

Case–II : Induced EMF in mutual induction

If current in primary coil (I_1) changes w.r.t. time, then



$$\frac{dI_1}{dt} \rightarrow \frac{dB_1}{dt} \rightarrow \frac{d\phi_1}{dt} \rightarrow \frac{d\phi_2}{dt}$$
$$N_2 \phi_2 = M I_1$$
$$-N_2 \frac{d\phi_2}{dt} = -M\frac{dI_1}{dt}$$
$$\boxed{e_2 = -M\left(\frac{dI_1}{dt}\right)}$$

$\frac{dI_1}{dt}$ → Rate of change of current in primary coil

e_2 → Induced emf in secondary coil

HENRY CLASSES

Illustration 25:

A solenoid has 2000 turns wound over a length of 0.3 m. The area of cross-section is 1.2×10^{-3} m^2. Around its central section a coil of 300 turns is closely wound. If an initial current of 2A is reversed in 0.25 s, find the emf induced in the coil.

Solution:

$$M = \frac{\mu_0 N_1 N_2 A}{\ell} = \frac{4\pi \times 10^{-7} \times 2000 \times 300 \times 1.2 \times 10^{-3}}{0.3} = 3 \times 10^{-3} H$$

$$E = -M\frac{\Delta I}{\Delta t} = -3 \times 10^{-3}\left[\frac{-2-2}{0.25}\right] = 48 \times 10^{-3} V = 48 \text{ mV}$$

Illustration 26:

On a cylindrical rod two coils are wound one above the other. What is the coefficient of mutual induction if the inductance of each coil is 0.1H?

Solution:

One coil is wound over the other and coupling is tight, so K = 1,

$$M = \sqrt{L_1 L_2} = \sqrt{0.1 \times 0.1} = 0.1H$$

Illustration 27:

How does the mutual inductance of a pair of coils change when :

(i) the distance between the coils is increased?

(ii) the number of turns in each coil is decreased?

(iii) a thin iron rod is placed between the two coils, other factors remaining the same?

Justify your answer in each case.

Solution:

(i) The mutual inductance of two coils, decreases when the distance between them is increased. This is because the flux passing from one coil to another decreases.

(ii) Mutual inductance $M = \frac{\mu_0 N_1 N_2 A}{\ell}$ i.e., $M \propto N_1 N_2$

Clearly, when the number of turns N_1 and N_2 in the two coils is decreased, the mutual inductance decreases.

(iii) When an iron rod is placed between the two coils the mutual inductance increases, because $M \propto$ permeability (μ)

Illustration 28:

A coil is wound on an iron core and looped back on itself so that the core has two sets of closely would wires in series carrying current in the opposite sense. What do you expect about its self-inductance? Will it be larger or small?

Solution:

As the two sets of wire carry currents in opposite directions, their induced emf's also act in opposite directions. These induced emf's tend to cancel each other, making the self-inductance of the coil very small.

HENRY CLASSES

BEGINNER'S BOX-6

1. The mutual inductance between a primary and secondary circuits is 0.5H. The resistance of the primary and the secondary circuits are 20Ω and 5 Ω respectively. To generate a current of 0.4 A in the secondary, current in the primary must be changed at the rate of :–

(1) 4.0 A/s (2) 16.0 A/s (3) 1.6 A/s (4) 8.0 A/s

2. Two coils A and B having turns 300 and 600 respectively are placed near each other, on passing a current of 3.0 ampere in A, the flux linked with A is 1.2×10^{-4} weber and with B it is 9.0×10^{-5} weber. The mutual inductance of the system is :-

(1) 2×10^{-5} H (2) 3×10^{-5} H (3) 4×10^{-5} H (4) 6×10^{-5} H

3. If the current in a primary circuit is $I = I_o \sin \omega t$ and the mutual inductance is M, then the value of induced voltage in secondary circuit will be :-

(1) $e = MI_o\omega \cos \omega t$ (2) $e = -MI_0\omega \cos \omega t$ (3) $e = [M\omega \cos \omega t]/I_o$ (4) $e = -(M\omega \cos \omega t)/I_o$

4. An a.c. of 50 Hz and 1 A peak value flows in primary coil transformer whose mutual inductance is 1.5 H. Then peak value of induced emf in secondary is :-

(1) 150 V (2) 150 π V (3) 300 V (4) 200 V

5. The number of turn of primary and secondary coil of a transformer is 5 and 10 respectively and the mutual inductance is 25 H. If the number of turns of the primary and secondary is made 10 and 5, then the mutual inductance of the coils will be :-

(1) 6.25 H (2) 12.5 H (3) 25 H (4) 50 H

6. The length of a solenoid is 0.3 m and the number of turns is 2000. The area of cross-section of the solenoid is $1.2 \times 10^{-3} m^2$. Another coil of 300 turns is wrapped over the solenoid. A current of 2A is passed through the solenoid and its direction is changed in 0.25 sec. then the induced emf in coil :-

(1) 4.8×10^{-2} V (2) 4.8×10^{-3} V (3) 3.2×10^{-4} V (4) 3.2×10^{-2} V

7. Two conducting loops of radi R_1 and R_2 are concentric and are placed in the same plane. If $R_1 > R_2$, the mutual inductance M between them will be directly proportional to :-

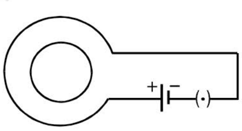
(1) R_1/R_2 (2) R_2/R_1 (3) R_1^2/R_2^2 (4) R_2^2/R_1

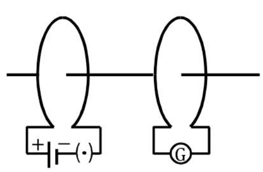
8. Find direction of induced current in secondary circuit for the following changes in primary circuit :–

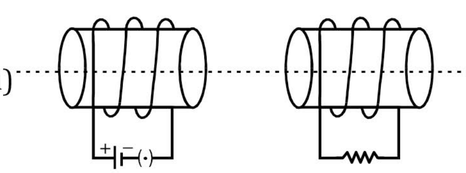
(a) Key is just closed

(b) Some time after the closing of key

(c) Key is just opened

(i) 

(ii) 

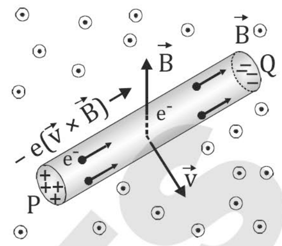
(iii) 

HENRY CLASSES

Dynamic EMI

Dynamic or Motional EMF

A conductor PQ is placed in a uniform magnetic field B, directed normal to the plane of paper outwards. PQ is moved with a velocity v, the free electrons of PQ also move with the same velocity. The electrons experience a magnetic Lorentz force $F_m = -e(\vec{v} \times \vec{B})$.



According to Fleming's left-hand rule, this force acts in the direction PQ and hence the free electrons will move towards Q. A negative charge accumulates at Q and a positive charge at P.

An electric field E is setup in the conductor from P to Q. Force exerted by electric field on the free electrons is, $\vec{F}_e = -e\vec{E}$

The accumulation of charge at the two ends continues till these two forces balance each other.

so $\quad \vec{F}_m = -\vec{F}_e \quad \Rightarrow \quad e(\vec{v} \times \vec{B}) = -\ e\vec{E} \quad \Rightarrow \quad \vec{E} = -(\vec{v} \times \vec{B})$

The potential difference between the ends P and Q is V = $-\ \vec{E}.\vec{\ell}\ = (\vec{v} \times \vec{B}).\vec{\ell}$. It is the magnetic force on the moving free electrons that maintains the potential difference and produces the emf $\varepsilon = B\ell v$ (for $\vec{B} \perp \vec{v} \perp \vec{\ell}$)

As this emf is produced due to the motion of a conductor, so it is called a motional emf.

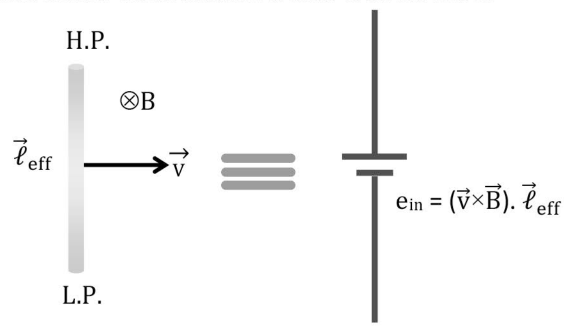
Important Points

- The concept of motional emf for a conductor can be generalized for any shape moving in any magnetic field uniform or not. For an element $\vec{d\ell}$ of conductor the contribution due to the emf is the magnitude $d\ell$ multiplied by the component of $\vec{v} \times \vec{B}$ parallel to $\vec{d\ell}$, that is $de = (\vec{v} \times \vec{B}).\vec{d\ell}$
- If $\vec{B} \perp \vec{v} \perp \vec{L}$, then $e_{in} = vBl_{eff}$
- If any two vectors among $\vec{B}, \vec{v}$ and $\vec{L}$ are parallel to each other then induced emf will be zero.
- For determining higher potential end, evaluate $\vec{v} \times \vec{B}$

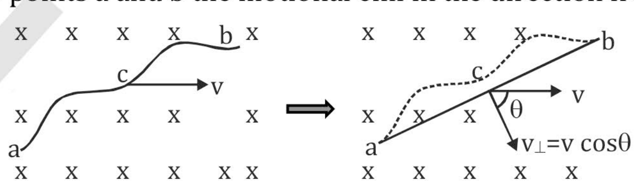
  Thumb : In the direction of motion

  Fingers : In the direction of magnetic field

  Palm : In the direction of High Potential



- For any two points a and b the motional emf in the direction from b to a is,



$$e = \int_b^a (\vec{v} \times \vec{B}).\vec{d\ell}$$

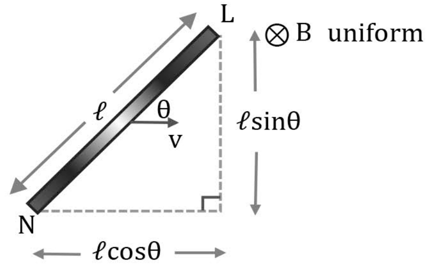
Motional emf in wire acb in a uniform magnetic field is the motional emf in an imaginary wire ab. Thus, $e_{acb} = e_{ab}$ = (length of ab) $(v_\perp)$ (B), $v_\perp$ = the component of velocity perpendicular to both $\vec{B}$ and ab. From right hand rule : b is at higher potential and a at lower potential.

Hence, $V_{ba} = V_b - V_a$ = (ab) (v cosθ) (B)

HENRY CLASSES

Example :

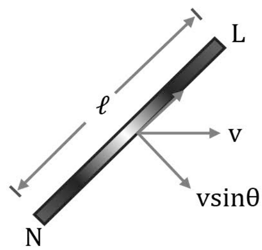
$$\left\{\begin{matrix} \vec{\ell}\ \theta\ \vec{v} \\ \vec{\ell} \perp \vec{B} \\ \vec{v} \perp \vec{B} \end{matrix}\right\}$$



$e_d = Bv(\ell \sin\theta)$

ℓ — $\ell\cos\theta \parallel v$, no flux cutting.
ℓ — $\ell\sin\theta \perp v$, do flux cutting.

OR

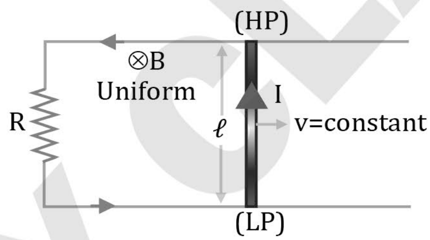


$e_d = B(v\sin\theta)\ell$

v — $v\cos\theta \parallel \ell$, no flux cutting.
v — $v\sin\theta \perp \ell$, do flux cutting.

Motion of straight conductor in horizontal plane

For the given circuit, if metal rod moves with uniform velocity 'v' by an external agent.



Induced emf in circuit $e = Bv\ell$

Current flows through circuit $I = \frac{e}{R} = \frac{Bv\ell}{R}$

Retarding opposing force exerted on metal rod by action of induced current

$$\vec{F}_m = I(\vec{\ell} \times \vec{B})$$

$$F_m = BI\ell$$

where $\theta = 90°$

$$F_m = \frac{B^2\ell^2 v}{R}$$

An external mechanical force (F_{ext}) is required for uniform velocity of metal rod.

For constant velocity, resultant force on metal rod must zero and for that $F_{ext} = F_m$

$$F_{ext.} = F_m = \frac{B^2\ell^2 v}{R}$$

If $(B, \ell, R) \rightarrow$ const. $\Rightarrow \quad F_{ext.} \propto v$

Hence, Vba = Vb – Va = (ab) (v cosθ) (B)

HENRY CLASSES

For uniform motion of metal rod, the rate of doing mechanical work by external agent or mechanical power delivered by external source given as :-

$$P_{mech} = P_{ext} = \vec{F}_{ext.}.\vec{v} = F_{ext}v$$

$$P_{ext.} = P_m = \frac{B^2\ell^2v^2}{R}$$

If $(B, \ell, R) \rightarrow$ const. $\Rightarrow$ $P_{mech.} \propto v^2$

Rate of heat dissipation across resistance or thermal power developed across resistance is :-

$$P_{th} = I^2R = R\left(\frac{Bv\ell}{R}\right)^2$$

$$\Rightarrow \quad P_{th} = \frac{B^2\ell^2v^2}{R}$$

It is clear that $P_{th} = P_{mech}$ which is consistent with the principle of conservation of energy.

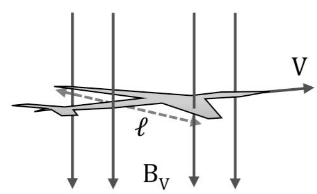
Illustration 29:

An aircraft with a wing span of 40 m flies with a speed of 1080 km h^{-1} in the eastward direction at a constant altitude in the northern hemisphere, where the vertical component of earth's magnetic field is 1.75×10^{-5} T. Find the emf that develops between the tips of the wings.

Solution:

The metallic part between the wing-tips can be treated as a single conductor cutting flux-lines due to vertical component of earth's magnetic field. So emf is induced between the tips of its wings.

Here $\ell = 40$ m, $B_V = 1.75 \times 10^{-5}$ T

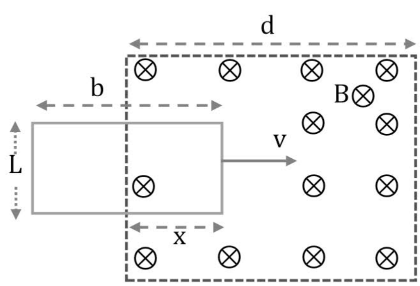


$$v = 1080\,kmh^{-1} = \frac{1080 \times 1000}{3600} ms^{-1} = 300\ ms^{-1}$$

$\therefore$ $E = B_V \ell v = 1.75 \times 10^{-5} \times 40 \times 300 = 0.21$ V

Illustration 30:

Figure shows a rectangular conducting loop of resistance R, width L, and length b being pulled at constant speed v through a region of width d in which a uniform magnetic field B is set up by an electromagnet.



(Let L = 40 mm, b = 10 cm, d = 15 cm, R = 1.6 Ω, B = 2.0 T and v = 1.0 m/s)

(i) Plot the flux (ϕ) through the loop as a function of the position (x) of the right side of the loop.

(ii) Plot the induced emf as a function of the position of the loop.

Solution:

(i) When the loop is not in the field : The flux linked with the loop $\phi = 0$

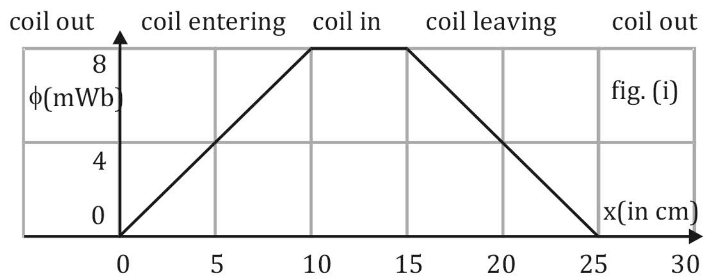
When the loop is entirely in the field : Magnetic flux linked with the loop

$\phi = BLb = 2 \times 40 \times 10^{-3} \times 10 \times 10^{-2} = 8$ mWb

When the loop is entering the field : The flux linked with the loop $\phi = BLx$

When the loop is leaving the field : The flux $\phi = BL[b - (x - d)]$

HENRY CLASSES



(ii) Induced emf is $e = -\frac{d\phi}{dt} = -\frac{d\phi}{dx}\frac{dx}{dt} = -\frac{d\phi}{dx}v$ = – slope of the curve of figure (i) × v

The emf for 0 to 10 cm :

$$e = -\frac{(8-0)\times 10^{-3}}{(10-0)\times 10^{-2}}\times 1 = -80\text{ mV}$$

The emf for 10 to 15 cm : e = 0 × 1 = 0

The emf for 15 to 25 cm :

$$e = -\frac{(0-8)\times 10^{-3}}{(25-15)\times 10^{-2}}\times 1 = +80\text{ mV}$$

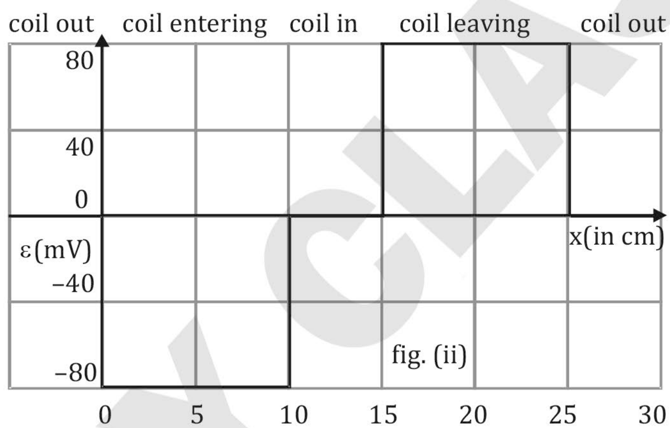
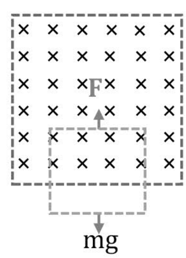


Illustration 31:

A horizontal magnetic field B is produced across a narrow gap between square iron pole-pieces as shown. A closed square wire loop of side ℓ, mass m and resistance R is allowed of fall with the top of the loop in the field. Show that the loop attains a terminal velocity given by $v = \frac{Rmg}{B^2\ell^2}$ while it is between the poles of the magnet.

Solution:

As the loop falls under gravity, the flux passing through it decreases and so an induced emf is set up in it. Then a force F which opposes its fall. When this force becomes equal to the gravity force mg, the loop attains a terminal velocity v.



The induced emf $e = Bv\ell$, and the induced current is $i = \frac{e}{R} = \frac{Bv\ell}{R}$

The force experienced by the loop due to this current is $F = B\ell i = \frac{B^2 v\ell^2}{R}$

When v is the terminal (constant) velocity F = mg

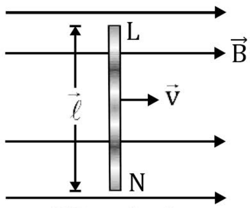
or $\frac{B^2 v\ell^2}{R} = mg$ or $v = \frac{Rmg}{B^2\ell^2}$

HENRY CLASSES

BEGINNER'S BOX-7

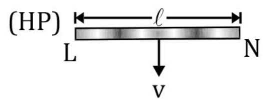
1. Find the Given Parameter when straight conductor moves in external magnetic field :–

(i)



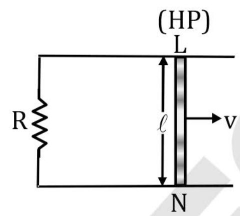
HP end = ?

(ii)



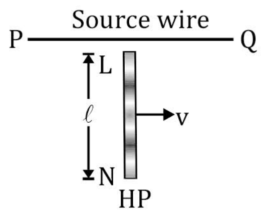
Direction of ext. B = ?

(iii)



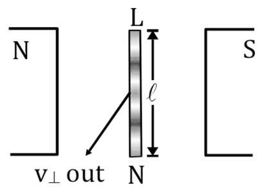
Direction ext. B = ?

(iv)



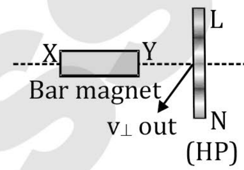
Direction of current in source wire PQ = ?

(v)



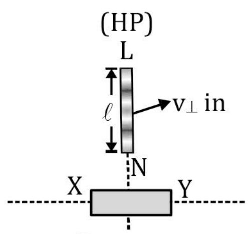
LP end = ?

(vi)



Poles of magnet=?

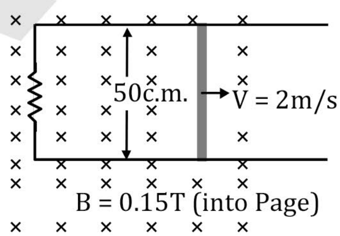
(vii)



Bar magnet

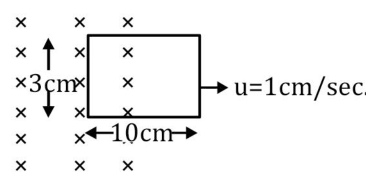
Poles of magnet = ?

2. A metallic rod completes its circuit as shown in the figure. The circuit is normal to a magnetic field of B = 0.15 T. If the resistance of the circuit is 3Ω the force required to move the rod with a constant velocity of 2m/sec. is:



(1) 3.75×10^{-3} N (2) 3.75×10^{-2} N (3) 3.75×10^{2} N (4) 3.75×10^{-4} N

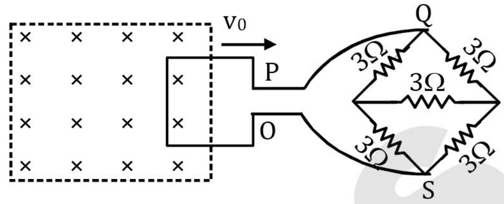
3. A rectangular loop sides 10 cm and 3cm moving out of a region of uniform magnetic field of 0.5T directed normal to the loop.
If we want to move loop with a constant velocity 1 cm/sec. then required mechanical force is (Resistance of loop = 1mΩ) :-



(1) 2.25×10^{-3} N (2) 4.5×10^{-3} N
(3) 9×10^{-3} N (4) 1.25×10^{-3} N

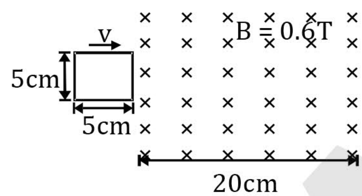
HENRY CLASSES

4. A metallic square wire loop of side 10 cm and resistance 1 Ω is moved with a constant velocity v_0 in a uniform magnetic field of induction B = 2 T as shown in the figure. The magnetic field perpendicular to the plane of the loop. The loop is connected to a network of resistors each of value 3 ohm. The resistance of the lead wires OS and PQ are negligible. What should be the speed of the loop so as to have a steady current of 1 mA in it? Give the direction of current in the loop.



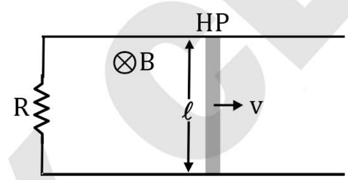
(1) 2×10^{-2} m/sec., anticlockwise direction (2) 4×10^{-2} m/sec., anticlockwise direction
(3) 2×10^{-2} m/sec., clockwise direction (4) 4×10^{-2} m/sec., clockwise direction

5. Figure shows a square loop of side 5 cm being moved towards right at a constant speed of 1cm/sec. The front edge just enters the 20 cm wide magnetic field at t = 0. Find the induced emf in the loop at t = 2s and t = 10s.



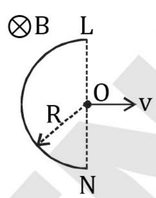
(1) 3×10^{-2}, zero (2) 3×10^{-2}, 3×10^{-4} (3) 3×10^{-4}, 3×10^{-4} (4) 3×10^{-4}, zero

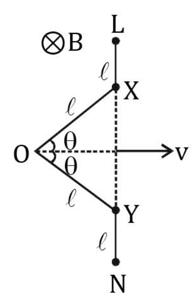
6. A conducting rod moves towards right with constant velocity v in uniform transverse magnetic field. Graph between force applied by the external agent v/s velocity and power supplied by the external agent v/s velocity.

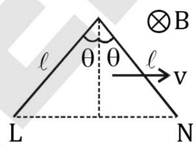


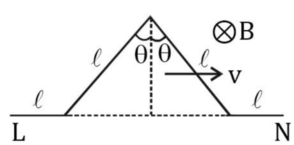
(1) St. line, parabola (2) Parabola, st. line (3) St. line, St. line (4) Parabola, Parabola

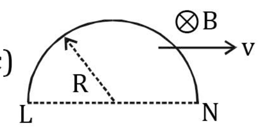
7. Find the induced EMF about ends of the rod in each case.

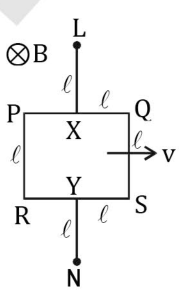
(i)


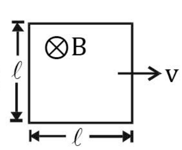
(ii)


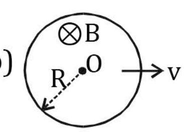
(iii) (a)


(b)


(c)


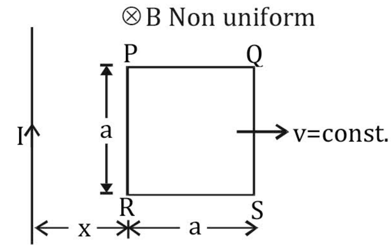
(iv)


(v) (a)


(b)


HENRY CLASSES

8. Find the EMF induced in metal loop when it moves in non-uniform magnetic field



Dynamic Induction due to Rotation of Rod

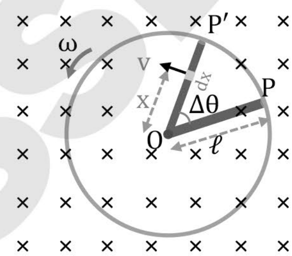
Let a conducting rod is rotating in a magnetic field around an axis passing through its one end, normal to its plane.

Consider an small element dx at a distance x from axis of rotation.

Suppose velocity of this small element = v

So, according to Lorentz formula induced e.m.f. across this small element

$d\varepsilon = Bv.dx$



∵ This small element dx is at distance x from O (axis of rotation)

∴ Linear velocity of this element dx is $v = \omega x$

Substitute of value of v in equation (i) $d\varepsilon = B\omega x dx$

Every element of conducting rod is normal to magnetic field and moving in perpendicular direction to the field

So, net induced e.m.f. across conducting rod $\varepsilon = \int d\varepsilon = \int_0^{\ell} B\omega x\, dx = \omega B\left(\frac{x^2}{2}\right)_0^{\ell}$

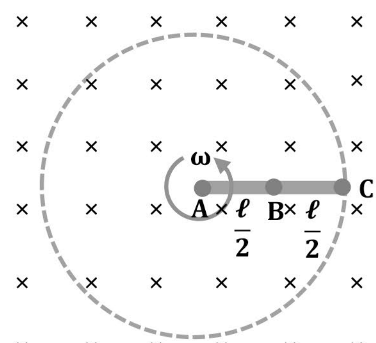
$\varepsilon = \frac{1}{2}B\omega\ell^2$ $\quad\varepsilon = \frac{1}{2}B\times 2\pi f\times\ell^2$ [f = frequency of rotation]

$= Bf\left(\pi\ell^2\right) = BAf$ (Where $A = \pi\ell^2$ i.e. area traversed by the rod)

Illustration 32:

Find potential difference between given cases.

(1) Between A and B (2) Between B and C (3) Between A and C



Solution:

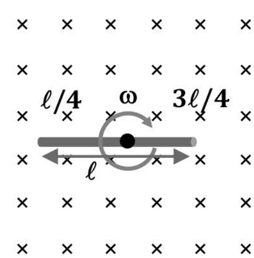
(1) $e_{AB} = VB\ell = (x\omega)Bdx = B\omega\int_0^{\ell/2} xdx = \frac{B\omega\ell^2}{8}$

(2) $e_{BC} = VB\ell = (x\omega)Bdx = B\omega\int_{\ell/2}^{\ell} xdx = \frac{3B\omega\ell^2}{8}$

(3) $e_{AC} = VB\ell = (x\omega)Bdx = B\omega\int_0^{\ell} xdx = \frac{B\omega\ell^2}{2}$

Illustration 33:

Find the induced emf across ends of rod.



Solution:

$e_{\ell/4} = (x\omega)Bdx = B\omega\int_0^{\ell/4} xdx \Rightarrow \frac{B\omega\ell^2}{32}$

$e_{3\ell/4} = (x\omega)Bdx = B\omega\int_0^{3\ell/4} xdx \Rightarrow \frac{9B\omega\ell^2}{32}$

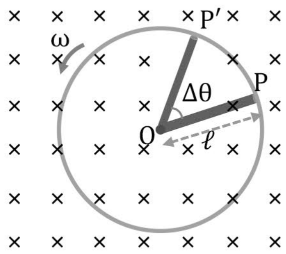
Induced emf across the road

$e = e_{3\ell/4} - e_{\ell/4} = \frac{B\omega\ell^2}{4}$

HENRY CLASSES

Dynamic Induction due to Rotation of Rod

A conducting rod of length ℓ whose one end is fixed, is rotated about the axis passing through it's fixed end and perpendicular to it's length with constant angular velocity ω. Magnetic field (B) is perpendicular to the plane of the paper.



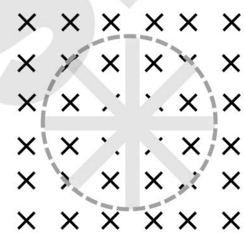
P.D. between center and end point

$$e=\frac{B\omega\ell^2}{2}$$

P.D. between two end points

e = 0

Dynamic Induction due to Rotation of Cycle Wheel Spokes



Due to flux cutting each metal spoke becomes identical cell of emf e (say), all such identical cells connected in parallel fashion e_{net} = e (emf of single cell)

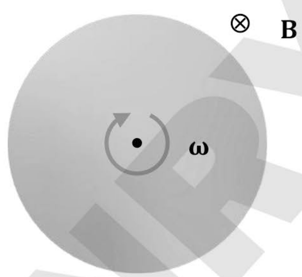
$$e_{net}=\frac{B\omega\ell^2}{2}$$

$$\omega=2\pi f$$

Special Note : - This emf does not depends on number of spokes ('N') in wheel.

Dynamic Induction due to Rotation of Disc

A metal disc can be assumed to made of uncountable radial conductors when metal disc rotates in transverse magnetic field these radial conductors cuts away magnetic field lines and because of this flux cutting all becomes identical cells each of emf 'e' where $e=\frac{B\omega r^2}{2}$



P.D. between center and Circumference

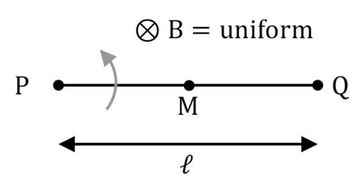
$$e=\frac{B\omega r^2}{2}$$

P.D. between two point on circumference

e = 0

Illustration 34:

A rod PQ of length ℓ is rotating about one end P in a uniform magnetic field B which is perpendicular to the plane of rotation of the rod. Point M is the mid point of the rod. Find the induced emf between M & Q if that between P & Q = 100V.



Solution:

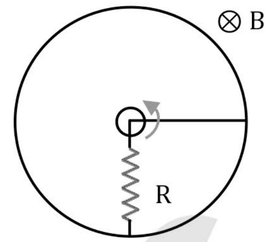
$E_{MQ} + E_{PM} = E_{PQ}$

$$E_{PQ}=\frac{Bw\ell^2}{2}=100$$

$$E_{MQ}+\frac{B\omega\left(\frac{\ell}{2}\right)^2}{2}=\frac{B\omega\ell^2}{2}\Rightarrow E_{MQ}=\frac{3}{8}B\omega\ell^2=\frac{3}{4}\times100V=75V$$

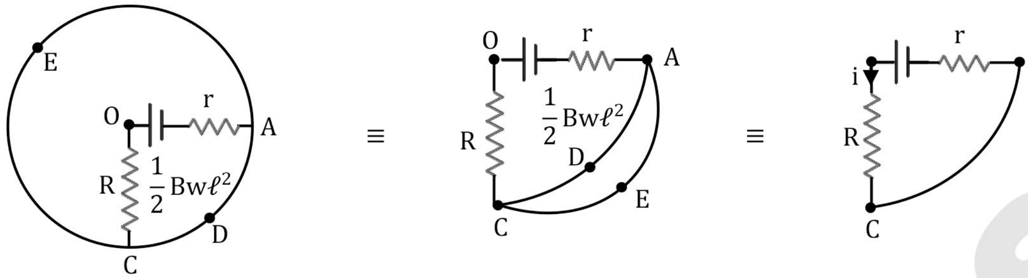
HENRY CLASSES

Illustration 35:



A rod of length ℓ and resistance r rotates about one end as shown in figure. Its other end touches a conducting ring of negligible resistance. A resistance R is connected between centre and periphery. Draw the electrical equivalence and find the current in the resistance R. There is a uniform magnetic field B directed as shown.

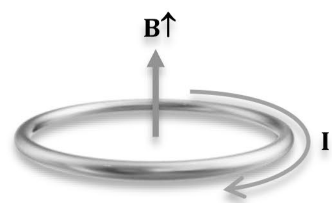
Solution:



$$\text{current } i = \frac{\frac{1}{2}B\omega\ell^2}{R+r}$$

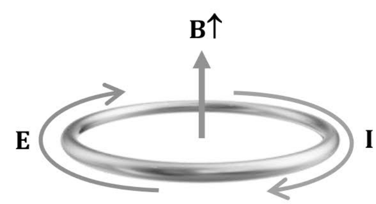
Induced Electric Field

Observations



1. When magnetic field changes with time in loop a current get induced in the loop.
2. This implies free electron (which were initially at rest) inside the loop experienced force due to which they constitute current.
3. Since only magnetic field is present therefore force must be magnetic (on charge at rest).
4. But we know that force on charge at rest in Magnetic field is always zero.
   $F = qvB$
   since v is zero, F must be zero.

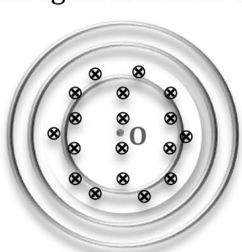
Conclusion



It was concluded by the scientist that since only electric field can apply force on charge at rest, a time varying magnetic field will produce an induce electric field, and this induce electric field will exert force on free electron at rest and current is produced.

Properties of Induce Electric field

1. When magnetic field changes with time in region then an electric field induces within and outside the region.



Concentric circular field lines of induced Electric field.

2. $\vec{F} = q\vec{E}$ is valid for this field.
3. This field is different from the conservative electrostatic field produced by stationary charges.
4. Direction of induced electric field is the same as direction of induced current.
5. Unlike electro-static field, these induced electric field lines always form closed loop and are non-conservative in nature.

Mathematical Analysis of Induce Electric field

For electrostatic field, $\oint \vec{E}\cdot d\vec{\ell} = 0$

For induced electric field, $\oint \vec{E}_{in}\cdot d\vec{\ell} \neq 0$

Definition of EMF : When a charges q goes once around the loop, the total work on it by the electric field per unit charge is equal to emf in loop

$$e = \frac{w_{loop}}{q} = \frac{\oint \vec{F}\cdot d\vec{\ell}}{q}$$

Now Induce EMF according to Faraday's Law is given by:

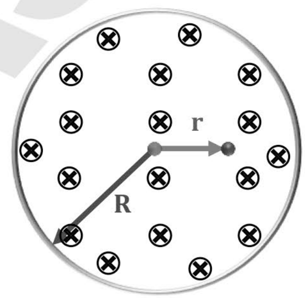
$$e = -\frac{d\phi}{dt} = \frac{w_{loop}}{q} = \frac{\oint \vec{F}\cdot d\vec{\ell}}{q}$$

$$-\frac{d\phi}{dt} = \frac{\oint q\vec{E}_{in}\cdot d\vec{\ell}}{q}$$

$$\oint \vec{E}_{in}\cdot d\vec{\ell} = -\frac{d\phi}{dt}$$

Calculation of induced electric field

1. Inside point (r < R)

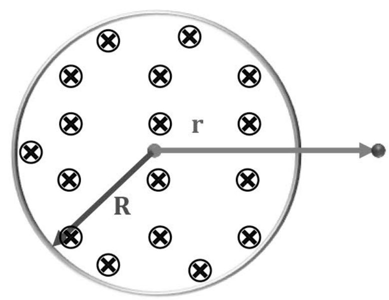


$$E(2\pi r) = \frac{AdB}{dt}$$

$$E(2\pi r) = \pi r^2\left(\frac{dB}{dt}\right)$$

$$E = \frac{r}{2}\frac{dB}{dt} \qquad (E_{inside} \propto r)$$

2. Outside point (r > R)

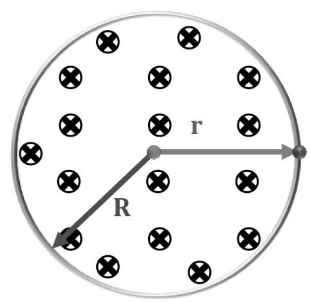


$$E(2\pi r) = A\left(\frac{dB}{dt}\right)$$

$$E(2\pi r) = \pi R^2\left(\frac{dB}{dt}\right)$$

$$E = \frac{R^2}{2r}\frac{dB}{dt} \qquad \left(E_{out} \propto \frac{1}{r}\right)$$

3. Surface point (r = R)



$$E(2\pi R) = \pi R^2\left(\frac{dB}{dt}\right)$$

$$E = \frac{R}{2}\frac{dB}{dt}$$

Variation of induced electric field with r

A cylindrical space of radius R is filled with an uniform magnetic induction B parallel to the axis of the cylinder. If B changes at a constant rate, the graph shown variation of induced electric field with distance r from the axis of cylinder will be like that

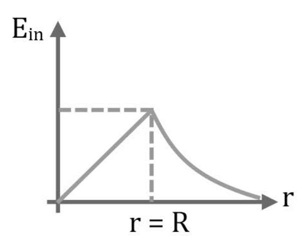


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Illustration 36:

A circular loop of radius 2cm, is placed in a time varying magnetic field with rate of 2T/sec. Then induced electric field in this loop will be :-

Solution:

$$E(2\pi R) = \pi R^2 \frac{dB}{dt} \qquad \Rightarrow \qquad E = \frac{R}{2}\frac{dB}{dt} = 1 \times 10^{-2} \times 2 = 0.02 \text{ V/m}$$

Illustration 37:

A 4 cm diameter solenoid is wound with 2000 turns per meter. The current through the solenoid oscillates at 60 Hz with an amplitude of 2 A. What is the maximum strength of the induced electric field inside the solenoid?

Solution:

Magnetic field strength inside a solenoid with n turns per meter is $B = \mu_0 nI$. In this case, the current through the solenoid is $I = I_0 \sin \omega t$, where $I_0 = 2$ A is the peak current and $\omega = 2\pi(60\text{Hz}) = 377$ rad/s. Thus, the induced electric field strength at radius r is

$$E = \frac{r}{2}\left|\frac{dB}{dt}\right| = \frac{r}{2}\frac{d}{dt}\left(\mu_0 n I_0 \sin\omega t\right) = \frac{1}{2}\mu_0 n r \omega I_0 \cos\omega t$$

The field strength is maximum at maximum radius (r = R) and at the instant when $\cos \omega t = 1$. That is,

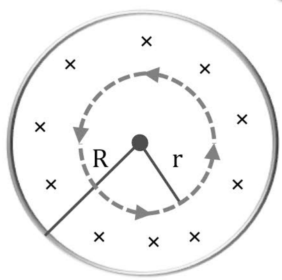
$$E_{max} = \frac{1}{2}\mu_0 n R \omega I_0 = 0.019 \text{V/m}$$

Illustration 38:

The magnetic field at all points within the cylindrical region whose cross-section is indicated in the figure start increasing at a constant rate α T/s. Find the magnitude of electric field as a function of r, the distance from the geometric centre of the region.

Solution:

For $r \le R$:



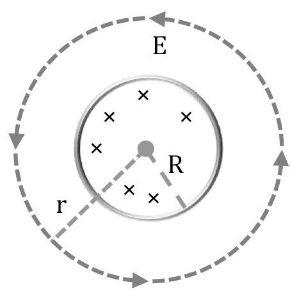
$\because \qquad E\ell = A\left|\frac{dB}{dt}\right|$

$\therefore \qquad E(2\pi r) = (\pi r^2)\alpha \Rightarrow E = \frac{r\alpha}{2} \Rightarrow E \propto r$

E-r graph is straight line passing through origin.

At $\qquad r = R,\ E = \frac{R\alpha}{2}$

For $\qquad r \ge R$:



$\because \qquad E\ell = A\left|\frac{dB}{dt}\right|$

$\therefore \qquad E(2\pi r) = (\pi R^2)\alpha$

$\Rightarrow \qquad E = \frac{\alpha R^2}{2r} \Rightarrow E \propto \frac{1}{r}$

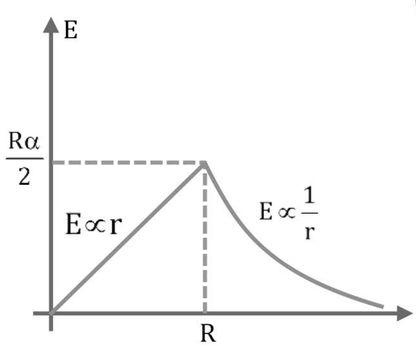


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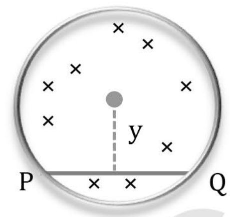
Illustration 39:

For the situation described in figure the magnetic field changes with time according to B = kt. Find the value of induced emf across PQ.



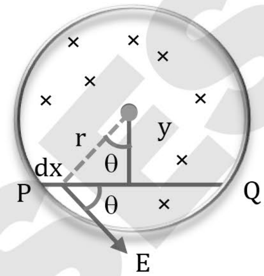
Solution:

M-1 :

$$\text{emf} = \vec{E}.d\vec{x}$$

$$\text{emf} = \frac{k}{2}(r.\cos\theta).dx \quad \Rightarrow \quad \int \text{emf} = \int \frac{ky}{2}dx = \frac{ky}{2} \times \ell$$

$$E_{PQ} = \frac{ky\ell}{2}$$



M-2

$$\phi_{OPQ} = B \times \frac{1}{2}(y\ell)$$

$$E_{OPQ} = \frac{d\phi}{dt} = \frac{ky\ell}{2}$$

$$E_{OP} = +E_{PQ} + E_{QO} = \frac{ky\ell}{2}$$

$$E_{PQ} = \frac{ky\ell}{2} \qquad (\because E_{OP} = E_{QO} = 0)$$

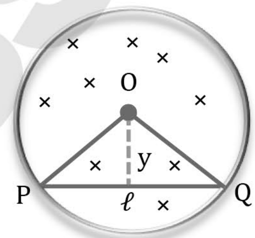
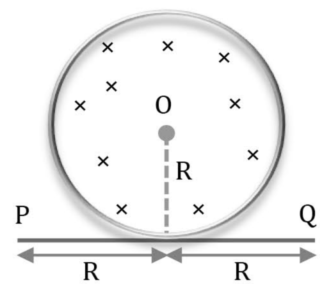


Illustration 40:

For the situation described in figure the magnetic field changes with time according to B = kt. Find the value of induced electric field across PQ.



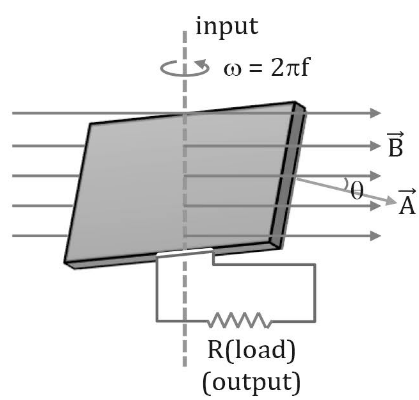
Solution:

$$E = \frac{d\phi}{dt} \Rightarrow E = \frac{dB}{dt} \times A$$

$$E_{OP} + E_{PQ} + E_{QO} = k \times \frac{\pi R^2}{4}$$

Periodic EMI

When a coil, which is placed in uniform magnetic field, rotates with constant angular frequency about shown axis then magnetic flux through the coil changes periodically with respect to time so an emf of periodic nature induced in coil. This phenomenon known as periodic emi.



- Magnetic flux through the rotating coil at any instant 't' :-

  $\phi = NBA\cos\theta = NBA\cos\omega t$ (as $\theta = \omega t$)

  $\phi = \phi_0 \cos\omega t$ where $\phi_0 = NBA$ is flux amplitude or max. flux

Sp. Note :– Magnetic flux changes periodically with respect to time.

- Induced emf in rotating coil at any instant 't' :-

  $$e = -\frac{d\phi}{dt} = NBA\omega \sin\omega t$$

  $e = e_0 \sin\omega t$ where $e_0 = NBA\omega = \phi_0\omega$ is emf amplitude or max. emf

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- Induced current in load circuit at any instant 't' :-

$$I = \frac{e}{R} = \frac{e_0}{R}\sin\omega t$$

$I = I_0 \sin\omega t$ where $I_0 = \frac{e_0}{R} = \frac{NBA\omega}{R} = \frac{\phi_0\omega}{R}$ is current Amplitude or max. current

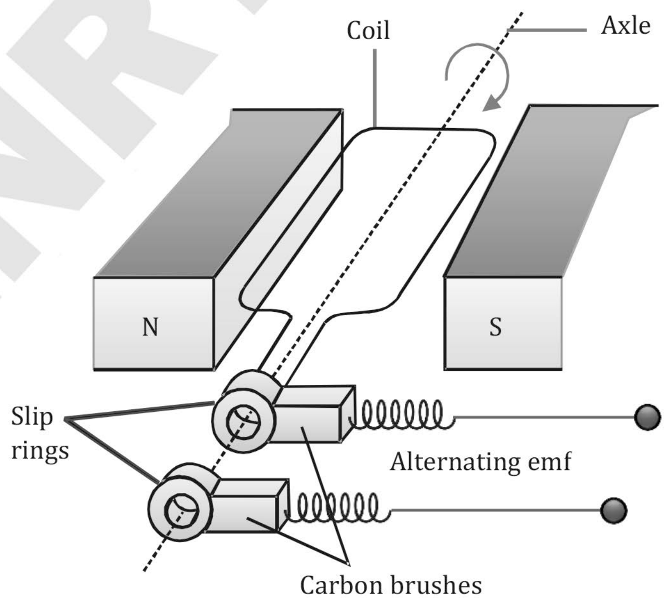
- Induced emf also changes in periodic manner that's why this phenomenon called periodic EMI.
- Phase difference between magnetic flux through the coil and induced emf is 90°.
  (a) When plane of coil perpendicular to $\vec{B} \Rightarrow \phi_{max}$ and $e_{min} = 0$
  (b) When plane of coil parallel to $\vec{B} \Rightarrow \phi_{min} = 0$ and e_{max}
- Induced emf and current acquires their max and min values simultaneously i.e. phase difference between both induced parameters is zero.

Phase (ωt)	$\phi = \phi_{max}\cos(\omega t)$	$E = e_{max}\sin\omega t$
$\omega t = 0$	$\phi = \phi_{max} = NBA$	$e = 0$
$\omega t = \frac{\pi}{2}$	$\phi = 0$	$e = e_{max} = NBA\omega$
$\omega t = \pi$	$\phi = \phi_{max} = -NBA$	$e = 0$
$\omega t = \frac{3\pi}{2}$	$\phi = 0$	$e = e_{max} = -NBA\omega$
$\omega t = 2\pi$	$\phi = \phi_{max} = NBA$	$e = 0$

- Frequency of induced parameter = Rotational frequency of coil = f.
- Induced emf and current changes their value with respect to time according to sine function, hence they called as sinusoidal induced quantities

Main Applications of EMI

(A) Generator (or Dynamo) :–



AC generator

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(i) **Work :-** It converts mechanical energy into electrical energy.

(ii) **Working principle** $\rightarrow$ Periodic E.M.I.

(iii) Types of Generator (According to output)
- → A.C. Generator
- → D.C. Generator

(iv) Generator has basic three sections

(a) Armature circuit (Internal circuit)

(b) Conveyor system (Connector of two circuit)

(c) Load circuit (External circuit)

(v) Basic difference between A.C.G. and D.C.G. in conveyor system.

For A.C. In conveyor system
- → Slip
- → Electric

For D.C. In conveyor system
- → Split Rings (Commutator)
- → Electric

Special chart For Rotating coil

Phy. Parameter	Equation	Max value	Frequency ($\phi = \omega/2\pi$)
(a) Magnetic flux	$\phi = \phi_0 \cos\omega t$	$\phi_0 = NBA$	f
(b) Induced emf	$e = e_0 \sin\omega t$	$e_0 = NBA\omega$	f
(c) Induced current	$I = I_0 \sin\omega t$	$I_0 = \frac{NBA\omega}{R}$	f

Losses in AC Generator

1. Cu Loss 2. Flux Leakage Loss 3. Iron Loss 4. Mechanical Loss

1. Cu Loss

These are associated with the I^2R loss in copper winding of armature coil.

2. Flux Leakage Loss

The useful (or main) flux is that which effectively links between magnet and armature. In practice, some of the flux will escape, or otherwise fail to link properly and will constitute flux linkage loss.

3. Iron loss

This loss takes place in soft iron core of armature. It is of two types:

(a) magnetic hysteresis loss in the iron core

(b) eddy current losses in the iron core.

4. Mechanical loss

This loss takes place due to friction between moving parts

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Eddy currents

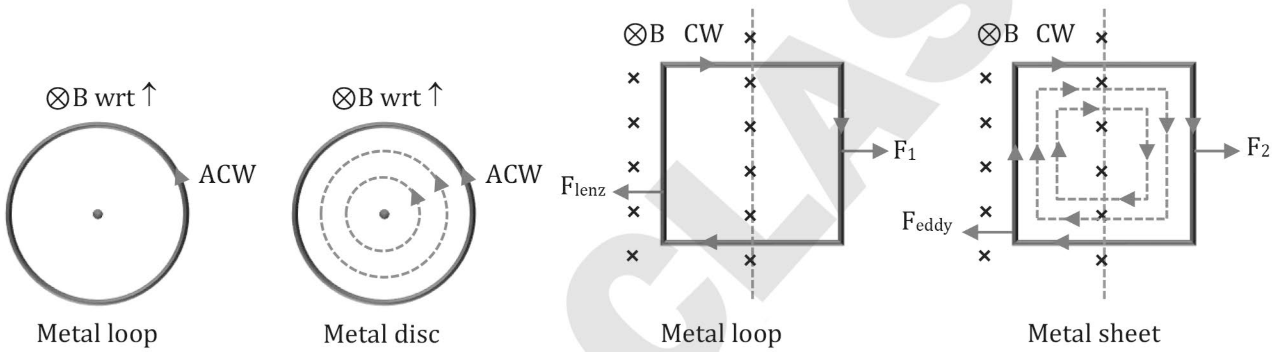
Eddy currents are the current induced in the body of a conductor when the magnetic flux linked with the conductor changes.

It is a group of induced currents which are produced, when metal bodies placed in time varying magnetic field or they moves in external magnetic field in such a way that flux through them changes with respect to time.

- Some of the applications of eddy currents are : Electromagnetic damping, induction furnace, electromagnetic brakes, induction motor, dead beat galvanometer, speedometers and in diathermy (deep heat treatment of parts of human body).
- Some of the undesirable effects of eddy currents are that they oppose the relative motive, involve loss of energy in the form of heat and reduce the life of electrical devices. To minimize eddy currents, laminated cores are used in a transformer.

Eddy current losses :

Eddy currents (or Focalt's currents) [Experimental verification by focalt]



($F_1 << F_2$)

Illustration 41:

A circular coil of radius 8.0 cm and 20 turns rotates about its vertical diameter with an angular speed of 50 s^{-1} in a uniform horizontal magnetic field of magnitude 3×10^{-2} T. Obtain the maximum and average induced emf in the coil. If the coil forms a closed loop of resistance 10 W, how much power is dissipated as heat? What is the source of this power?

Solution:

Induced emf in coil :-

$e = NBA\omega \sin\omega t$

$e_{max} = NBA\omega = NB(\pi r^2)\omega$

$e_{max} = 20 \times 3.0 \times 10^{-2} \times \pi \times 64 \times 10^{-4} \times 50 = 0.603$ V

e_{avg} is zero over a one cycle

$$I_{max} = \frac{e_{max}}{R} = \frac{0.603}{10} = 0.0603\text{ A}$$

$$P_{avg} = \frac{I_{max}^2 R}{2} = 0.018\text{W}$$

The induced current causes a torque opposing the rotation of the coil. An external agent (rotor) must supply torque (and do work) to counter this torque in order to keep the coil rotating uniformly. Thus, the source of the power dissipated as heat in the coil is the external rotor.

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Illustration 42:

In order to avoid eddy currents in the core of a transformer :-

(1) The number of turns in the secondary coil is made considerably large

(2) A laminated core is used

(3) A step down transformer is used

(4) A high voltage alternating weak current is used

Solution:

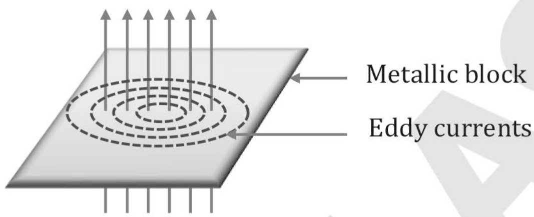
A laminated core is used so that eddy current minimized.

Illustration 43:

Plane of eddy currents makes an angle with the plane of magnetic lines of force equal to :-

(1) 40° (2) 0° (3) 90° (4) 180°

Solution:



Direction of eddy currents is given by Lenz's law. Hence, angle between eddy currents plane and magnetic field plane lines will be 90°.

Illustration 44:

The working of dynamo is based on principle of :-

(1) Electromagnetic induction (2) Conversion of energy into electricity

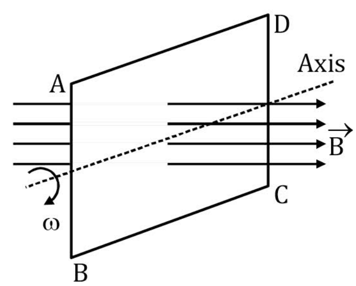
(3) Magnetic effects of current (4) Heating effects of current

Solution:

Electromagnetic induction

BEGINNER'S BOX-8

1. A rectangular coil ABCD is rotated in uniform magnetic field with constant angular velocity about its one of the diameter as shown in figure. The induced emf will be maximum, when the plane of the coil is :-



(1) Perpendicular to the magnetic field

(2) Making an angle of 30° with the magnetic field.

(3) making an angle of 45° with the magnetic field.

(4) Parallel to the magnetic field.

2. A rectangular coil has 60 turns and its length and width is 20 cm and 10 cm respectively. The coil rotates at a speed of 1800 rotation per minute in a uniform magnetic field of 0.5 T about its one of the diameter. The maximum induced emf will be :-

(1) 98 V (2) 110 V (3) 113 V (4) 118 V

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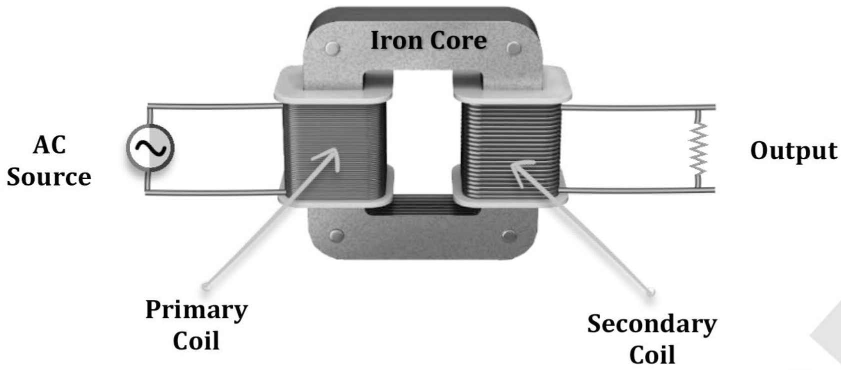
Transformer



(i) **Working principle :–** Mutual induction

(ii) **Transformer has basic two section** :-

(a) **Shell :-** It consist of primary and secondary coils of copper. The effective resistance between primary and secondary coil is infinite because electric circuit between two is open ($R_{ps} = \infty$)

(b) **Core :-** Both Cu coils are tightly wound over a bulk metal piece of high magnetic permeability (eg. soft iron) called core. Both coils are electrically insulated to core but core part magnetically coupled to both the coils.

(iii) **Work :–** It regulates A.C. voltage and transfers the electrical power without change in frequency of input supply. (The alternating current changes itself.)

(iv) **Special Points :-**

- It can't work with D.C. supply. If a battery is connected to its primary then output is across secondary is always zero i.e. No working of transformer.
- It can't called 'Amplifier' as it has no power gain like **transistor**.
- It has no moving part hence there are no mechanical losses in transformer, so its efficiency is higher than generator and motor.

(v) **Types** (According to voltage regulation) :–

Type
- → Step up transformer (S.U.T), $N_S > N_P$
- → Step down transformer (S.D.T) $N_S < N_P$

(vi) **S.U.T.** ⇒ converts **low voltage, high current** in to **high voltage, low current**

S.D.T. ⇒ converts **high voltage, low current** into **low voltage, high current.**

(vii) Power transmission is carried out always at **"High voltage, low current"** so that voltage drop and power losses are minimum in transmission line.

Voltage drop $= I_L R_L$, I_L : Line current R_L : total line resistance,

$$I_L = \frac{\text{Power to be transmitted}}{\text{Line voltage}}$$

Power losses $= I_L^2 R_L$

(viii) Sending power always at high voltage & low current (By. S.U.T.) and receiving power always at low voltage & high current (By S.D.T.)

(ix) High voltage coil having more number of turns and always **made of thin wire** and **high current coil** having less number of turns and always **made of thick wires.**

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(x) Ideal Transformer : (η = 100%)

(a) No flux leakage :-

$$\phi_s = \phi_p = \frac{-d\phi_s}{dt} = \frac{-d\phi_p}{dt}$$

$e_s = e_p = e$ (induced emf per turn of each coil is also same)

total induced emf for secondary $E_s = N_s e$

total induced emf for primary $E_p = N_p e$

$$\frac{E_s}{E_p} = \frac{N_s}{N_p} = n \text{ or } p \qquad ...(1)$$

where

→ n : turn ratio

→ p : transformation ratio

(b) No load condition :-

$V_P = E_P$ & $E_S = V_S$

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} \qquad ...(2)$$

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} = n \text{ or } p \qquad ...(3) \text{ [from (1) \& (2)]}$$

(c) No power loss :-

$P_{out} = P_{in}$

$V_S I_S = V_P I_P$

$$\frac{V_S}{V_P} = \frac{I_P}{I_S} \qquad ...(4)$$

from equation. (3) & (4)

$$\boxed{\frac{V_S}{V_P} = \frac{I_P}{I_S} = \frac{N_S}{N_P} = n \text{ or } p}$$

Sp. Note : Generally transformers deals in ideal condition i.e. $P_{in} = P_{out}$, if other information are not given.

(xi) Real transformer ($\eta \neq$ 100%) :- Some power is always lost due to flux leakage, hysteresis, eddy currents, and heating of coils. hence $P_{out} < P_{in}$ always.

Efficiency of transformer $\boxed{\eta = \frac{P_{out}}{P_{in}} = \frac{V_S I_S}{V_P I_P} \times 100}$

Losses in Transformer :-

(i) Copper or joule heating losses :-

Where : These losses occurs in both coils of shell part.

Reason : Due to heating effect of current ($H = I^2Rt$).

Remedy : To minimise these losses, high current coil always made up with thick wire and for removal of produced heat, circulation of mineral oil should be used.

(ii) Flux leakage losses :-

Where : These losses occurs in between both the coil of shell part.

Cause : Due to air gap between both the coils.

Remedy : To minimise these losses both coils are tightly wound over a common soft iron core (high magnetic permeability) so a closed path of magnetic field lines formed itself within the core and tries to makes coupling factor $K \rightarrow 1$

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(iii) Iron losses :-

Where : These losses occurs in core part.

Types :
- (a) Hysteresis losses
- (b) Eddy currents losses

(a) Hysteresis losses :-

Cause : Transformer core always present in the effect of alternating magnetic field ($B = B_0 \sin\omega t$) so it will magnetised & demagnetised with very high frequency ($f = 50$ Hz). During its demagnetization a part of magnetic energy left inside core part in form of residual magnetic field. Finally, this residual energy waste as heat.

Remedy : To minimise these losses material of transformer core should be such that it can be easily magnetised & demagnetised. For this purpose, magnetic soft materials should be used.

Ex. : Soft Iron | Low retentivity, Low coercivity

(b) Eddy current losses :

It is a group of induced currents which are produced, when metal bodies placed in time varying magnetic field or they moves in external magnetic field in such a way that flux through them changes with respect to time.

Key Points

(i) These currents are produced only in closed path within the entire volume and on the surface of metal body. Therefore their measurement is impossible.

(ii) Circulation plane of these currents is always perpendicular to the external magnetic field direction.

(iii) Generally resistance of metal bodies is low so magnitude of these currents is very high.

(iv) These currents can heat up the metal body and some time body will melt out (Application : Induction furnace)

(v) Due to these induced currents a strong eddy force (or torque) acts on metal body which always opposes the translatory (or rotatory) motion of metal body, according to Lenz law.

(vi) **Cause :** Transformer core is always present in the effect of alternating magnetic field ($B = B_0 \sin\omega t$). Due to this eddy currents are produced in its volume, so a part of magnetic energy of core is wasted as heat.

Remedy : To minimise these losses transformer core should be laminated. with the help of lamination process, circulation path of eddy current is greatly reduced & net resistance of system is greatly increased. So, these currents become feeble.

Illustration 45:

In a transformer, 220 ac voltage is increased to 2200 volts. If the number of turns in the secondary are 2000, then the number of turns in the primary will be

(1) 200 (2) 100 (3) 50 (4) 20

Solution:

$$\frac{V_P}{V_S} = \frac{N_P}{N_S} \quad \Rightarrow \quad N_P = \frac{220}{2200} \times 2000 = 200$$

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Illustration 46:

A power transmission line feeds input power at 2300 V to a step-down transformer having 4000 turns in its primary. What should be the number of turns in the secondary to get output power at 230 V?

Solution:

$V_p = 2300$ V ; $N_P = 4000$, $V_S = 230$ V

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} \qquad \therefore\ N_S = N_P \times \frac{V_S}{V_P} = 4000 \times \frac{230}{2300} = 400$$

Illustration 47:

The output voltage of an ideal transformer, connected to a 240 V a.c. mains is 24 V. When this transformer is used to light a bulb with rating 24V, 24W calculate the current in the primary coil of the circuit.

Solution:

$V_P = 240$ V, $V_S = 24$ V,

$V_S I_S = 24$ W

Current in primary coil $I_P = \frac{V_S I_S}{V_P} = \frac{24}{240} = 0.1A$

Illustration 48:

Primary winding and secondary winding of a transformer has 100 and 300 turns respectively. If its input power is 60 W then output power of the transformer will be: -

(1) 240 W (2) 180 W (3) 60 W (4) 20 W

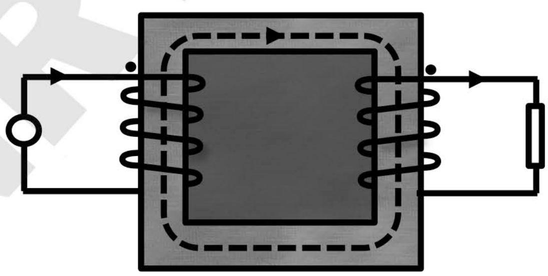
Solution:

In a transformer $\Rightarrow P_{input} = P_{output}$

Hence, output power of transformer will be 60 W.

Illustration 49:

A bulb (100 W, 110 V) is operated using a transformer by supply 220 V, 0.5 A, find efficiency of transformer.



Solution:

$P_{input} = P_{output}$

Efficiency $(\eta) = \frac{P_{output}}{P_{input}} \times 100$

Efficiency $(\eta) = \frac{100}{(220)(0.5)} \times 100$

Efficiency $(\eta) = 90.90\%$

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Illustration 50:

The primary and secondary coils of a transformer have 50 and 1500 turns, respectively. If the magnetic flux linked with primary coil is given by $\phi = 2 + 4t$ wb, then the output voltage across the secondary coil is:

(1) 30 V (2) 90 V (3) 120 V (4) 220 V

Solution:

$N_S = 1500$ $N_P = 50\ \phi = 2 + 4t$

$\frac{d\phi}{dt} = 4$ $\Rightarrow$ $e = V_i = 4$

$\frac{V_0}{V_i} = \frac{N_S}{N_P}$ $\Rightarrow$ $\frac{V_0}{4} = \frac{1500}{50}$

$V_0 = 120$ V

So, output voltage across the secondary coil is 120 V.

Illustration 51:

Input voltage of a transformer is 2500 volts and output current is 80 ampere. The ratio of turns in primary coil to secondary coil is 20 : 1. If efficiency of transformer is 100%, then find the voltage in secondary coil.

Solution:

$V_{input} = 2500$ $\Rightarrow$ $I_{output} = 80$

$\eta = 100\%$ $\Rightarrow$ $\frac{N_P}{N_S} = \frac{20}{1}$

$P_{input} = P_{output}$

$V_P I_P = V_S I_S$ $\left(\frac{V_S}{V_P} = \frac{N_S}{N_P} = \frac{I_P}{I_S}\right)$

$\frac{V_S}{2500} = \frac{1}{20}$ $\Rightarrow$ $V_S = 125$ V

So, output voltage across the secondary coil is 125 V.

BEGINNER'S BOX-9

1. Why the core of transformer is laminated?
2. A step-down transformer is used to reduce the main supply of 220 V to 11 V. If the primary coil draws a current of 5A and the current in secondary coil 90A, what is the efficiency of the transformer?
3. Why can't transformer be used to step up d.c. voltage?
4. Write two applications of eddy currents.

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BEGINNER'S BOX ANSWERS KEY

BEGINNER'S BOX-1

1. 5×10^{-3} Weber **2.** –NBA

3. Face ABCD $\Rightarrow +Ba^2$, Face EFGH $\Rightarrow -Ba^2$, Remaining faces $\Rightarrow$ Zero

4. 0.02 Wb **5.** –0.1 Wb **6.** 29.6×10^{-6} Wb

BEGINNER'S BOX-2

1. (i) Anticlockwise (ii) Clockwise (iii) A – Positive charge, B – Negative charge
(iv) Anticlockwise (v) Anticlockwise (vi) No induced current

(vii) (a) Anticlockwise

(b) Anti clockwise in bigger loop & clockwise in maller loop

(c) Anti clockwise in bigger loop & clockwise in smaller loop

(d) Anticlockwise in both loop & through connecting wire zero current

(viii) Anticlockwise

2. **(i)** **(a)** Anticlockwise (ACW), **(b)** Clockwise (CW)

(ii) N to L

(iii) Plate A – Positive charge, Plate B – Negative charge

BEGINNER'S BOX-3

1.	(4)	**2.**	(2)	**3.**	(4)	**4.**	(3)	**5.**	(2)
6.	(2)	**7.**	(1)	**8.**	(2)	**9.**	(2)		

BEGINNER'S BOX-4

1.	(3)	**2.**	(2)	**3.**	(1)	**4.**	(2)	**5.**	(1)
6.	(4)	**7.**	(4)						

BEGINNER'S BOX-5

1.	(4)	**2.**	(2)	**3.**	(1)	**4.**	(1)	**5.**	(4)
6.	(1)	**7.**	(4)	**8.**	(4)	**9.**	(2)		

BEGINNER'S BOX-6

1.	(1)	**2.**	(2)	**3.**	(2)	**4.**	(2)	**5.**	(3)
6.	(1)	**7.**	(4)						

8. (i) (a) ACW, (b) Zero, (c) CW

(ii) (a) ACW, (b) Zero, (c) CW

(iii) (a) L to N, (b) Zero, (c) N to L

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BEGINNER'S BOX-7

1. (i) No induced EMF (ii) B outwards (iii) B inwards

(iv) Direction of current Q to P (v) Low potential is N

(vi) Y south & X north (vii) X south; Y north

2. (1) **3.** (1) **4.** (3) **5.** (4) **6.** (1)

7. (i) 2 BvR, (ii) 2 Bvl (1 + sin θ), (iii) (a) Zero, (b) Zero, (c) Zero (iv) 3 Bvl (v) (a) Zero; (b) Zero

8. $e_{net} = \frac{\mu_0 I v a^2}{2\pi x(x+a)}$

BEGINNER'S BOX-8

1. (4) **2.** (3)

BEGINNER'S BOX-9

1. To reduce eddy current **2.** 90%

3. Working of transformer is based on mutual induction

4. Application of eddy current

(i) Induction furnace, (ii) Electric Brakes

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