Capacitors
THEORY

Capacitor is an arrangement of two conductors carrying charges of equal magnitudes and opposite sign and separated by an insulating medium. The following points may be carefully noted.

1. The net charge on the capacitor as a whole is zero. When we say that a capacitor has a charge Q, we mean that the positively charged conductor has a charge $+Q$ and negatively charged conductor has a charge $-Q$.

2. The positively charged conductor is at a higher potential than the negatively charged conductor. The potential difference V between the conductors is proportional to the charge magnitude Q and the ratio Q/V is known as *capacitance* C of the capacitor.

$$C = \frac{Q}{V}$$

Unit of capacitance is farad (F). The capacitance is usually measured in microfarad (μF).

$$1\ \mu F = 10^{-6}\ F$$

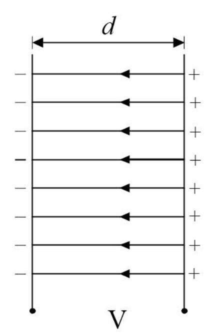
3. In a circuit, a capacitor is represented by the symbol : —||—

4. Capacitors work as a charge-storing or energy-storing devices. A capacitor can be thought of as a device which stores energy in the form of electric field. Energy stored in a capacitor is denoted by U. If V is the potential difference across the capacitor and Q is the charge on the capacitor and C is the capacitance of capacitor, then :

$$U = \frac{1}{2}CV^2 \quad \text{or} \quad U = \frac{1}{2}\frac{Q^2}{C} \quad \text{or} \quad U = \frac{1}{2}QV$$

Parallel Plate Capacitor :

The parallel plate capacitor consists of two metal plates placed parallel to each other and separated by a distance that is very small as compared to the dimension of the plates. The electric field between the plates is given by :



$$E = \frac{\sigma}{k\varepsilon_0}$$

where σ : surface charge density on either plate

k : dielectric constant of the medium between plates

If d is the separation between plates and A is the area of each plate, the potential difference (V) between plates is given as :

$$V = E\, d$$

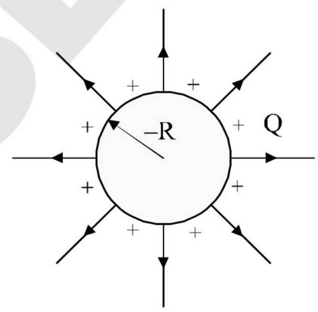
$$V = \frac{\sigma}{k\varepsilon_0} d = \frac{Q}{k\varepsilon_0 A} d$$

$$C = \frac{k\varepsilon_0 A}{d}$$ for parallel plate capacitor.

Note : If there is vacuum between the plates, $k = 1$.

Isolated Sphere as a Capacitor :

A conducting sphere of radius R carrying a charge Q can be treated as a capacitor with high-potential conductor as the sphere itself and the low-potential conductor as a sphere of infinite radius. The potential difference between these two spheres is :



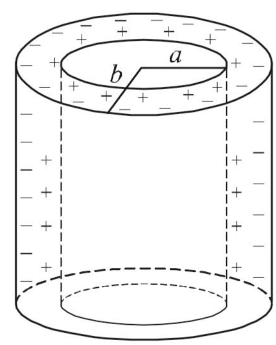
$$V = \frac{Q}{4\pi\,\varepsilon_0 R} - 0$$

$$\text{Capacitance } (C) = \frac{Q}{V}$$

$$C = 4\,\pi\,\varepsilon_0 R$$

Cylindrical Capacitor :

Cylindrical capacitor consists of two co-axial cylinders of radii a and b and length ℓ. The electric field exists in the region between the cylinders. Let k be the dielectric constant of the material between the cylinders. The capacitance is given by :



$$C = \frac{2\pi\, k\,\varepsilon_0\, \ell}{\ell og \frac{b}{a}}$$

Spherical Capacitor :

A spherical capacitor consists of two concentric spheres of radii a and b as shown. The inner sphere is positively charged to potential V and outer sphere is at zero potential. The inner surface of the outer sphere has an equal negative charge.

The potential difference between the spheres is :

$$V - 0 = \frac{Q}{4\pi\,\varepsilon_0\, a} - \frac{Q}{4\pi\,\varepsilon_0\, b} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$C = \frac{4\pi \varepsilon_0 ab}{b-a}$$

For a dielectric (k) between the spheres :

$$C = \frac{4\pi k \varepsilon_0 ab}{b-a}$$

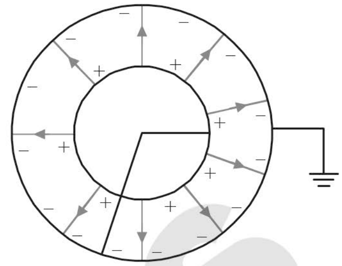


Illustration - 1 *The plates of a parallel plate capacitor are 5 mm apart and 2 m^2 in area. The plates are in vacuum. A potential difference of 10, 000 V is applied across the capacitor. Calculate :*

(a) *the capacitance,*

(b) *the charge on each plate,*

(c) *the electric field in space between the plates*

(d) *the energy stored in the capacitor.*

SOLUTION :

(a) $C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 2}{5 \times 10^{-3}}$

$C = 0.00354 \ \mu F.$

(b) $Q = CV = (0.00354 \times 10^{-6}) \times (10000)$

$Q = 3.54 \times 10^{-5}\ C = 35.4\ \mu C$

The plate at higher potential has a positive charge of magnitude 35.4 μC and the plate at lower potential has a negative charge of –35.4 μC.

(c) $E = \frac{V}{d} = \frac{10000}{5 \times 10^{-3}}\ V/m = 20 \times 10^5\ N/C$

or alternatively

$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} = \frac{3.54 \times 10^{-5}}{8.85 \times 10^{-12} \times 2}$

$= 20 \times 10^5\ N/C$

(d) $U = \frac{1}{2} CV^2$

$U = \frac{1}{2} \times (3.54 \times 10^{-9})\ (10000)^2 = 0.177\ J.$

Illustration - 2 *A parallel plate air capacitor is made using two plates 0.2 m square, spaced 1 cm apart. It is connected to a 50 – V battery.*

(a) *What is the capacitance ?*

(b) *What is the charge on each plate ?*

(c) *What is the energy stored in the capacitor ?*

(d) *What is the electric field between the plates ?*

(e) *If the battery is disconnected and then the plates are pulled apart to a separation of 2 cm, what are the answers to the above parts ?*

SOLUTION :

(a) $C_0 = \frac{\varepsilon_0 A}{d_0} = \frac{8.85 \times 10^{-12} \times 0.2 \times 0.2}{0.01}$

$C_0 = 3.54 \times 10^{-5}\ \mu F.$

(b) $Q_0 = C_0 V_0 = (3.54 \times 10^{-5} \times 50)\ \mu C$

$Q_0 = 1.77 \times 10^{-3}\ \mu C$

(c) $U_0 = 1/2\ C_0 V_0^2 = 1/2\ (3.54 \times 10^{-11})\ (50)^2$

$U_0 = 4.42 \times 10^{-8}\ J.$

(d) $E_0 = \frac{V_0}{d_0} = \frac{50}{0.01} = 5000\ V/m.$

(e) If the battery is disconnected, the charge on the capacitor plates remains constant while the potential difference between plates can change.

$$C = \frac{\varepsilon_0 A}{d} = \frac{1}{2}C_0 = 1.77 \times 10^{-5}\ \mu F.$$

$$Q = Q_0 = 1.77 \times 10^{-3}\ \mu C$$

$$V = \frac{Q}{C} = \frac{Q_0}{C_0/2} = 2V_0 = 100\ volts$$

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q_0^2}{(C_0/2)}$$

$$= 2U_0 = 8.84 \times 10^{-8}\ J$$

$$E = \frac{V}{d} = \frac{2V_0}{2d_0} = E_0 = 5000\ V/m.$$

Work has to be done against the attraction of plates when they are separated. This work gets stored in the energy of the capacitor.

Illustration - 3 *In the last example, suppose that the battery is kept connected while the plates are pulled apart. What are the answers to the parts (a), (b), (c) and (d) in that case ?*

SOLUTION :

If the battery is kept connected, the potential difference across the capacitor plates always remains equal to the emf of battery and hence is constant.

$$V = V_0 = 50\ \text{volts.}$$

$$C = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{2d_0} = \frac{1}{2}C_0 = 1.77 \times 10^{-5}\ \mu F$$

$$Q = CV = \frac{1}{2}C_0 V_0 = \frac{1}{2}Q_0$$

$$= 8.85 \times 10^{-4}\ \mu C.$$

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{1}{2}C_0\right)V_0^2 = \frac{1}{2}U_0$$

$$= 2.21 \times 10^{-8}\ J.$$

$$E = \frac{V}{d} = \frac{V_0}{2d_0} = \frac{1}{2}E_0 = 2500\ V/m$$

Note that the energy stored in the capacitor decreases. The work done to separate plates and the energy loss from capacitor get converted to heat dissipation in the connecting wire and energy used in charging the battery.

Illustration - 4 *A parallel plate capacitor has plates of area 4 m^2 separated by a distance of 0.5 mm. The capacitor is connected across a cell of emf 100 volts.*

(a) *Find the capacitance, charge amd energy stored in the capacitor.*

(b) *A dielectric slab of thickness 0.5 mm is inserted inside this capacitor after it has been disconnected from the cell. Find the answers to part (a) if $k = 3$.*

SOLUTION :

(a) $C_0 = \dfrac{\varepsilon_0 A}{d} = \dfrac{8.85 \times 10^{-12} \times 4}{0.5 \times 10^{-3}}$

$C_0 = 7.08 \times 10^{-2}\ \mu F.$

$Q_0 = C_0 V_0$

$= (7.08 \times 10^{-2} \times 100)\ \mu C = 7.08\ \mu C.$

$U_0 = \dfrac{1}{2} C_0 V_0^2 = 345 \times 10^{-6}\ J.$

(b) As the cell has been disconnected, $Q = Q_0$

$C = \dfrac{k\,\varepsilon_0 A}{d} = kC_0 = 0.2124 \mu F$

$V = \dfrac{Q}{C} = \dfrac{Q_0}{kC_0} = \dfrac{V_0}{k} = \dfrac{100}{3}$ volts.

$U = \dfrac{1}{2}\dfrac{Q_0}{C} = \dfrac{1}{2}\dfrac{Q_0^2}{kC_0} = \dfrac{U_0}{k}$

$= 118 \times 10^{-6}$ J.

Electric field inside the plates

$= E\ \dfrac{V}{d} = \dfrac{V_0}{kd} = \dfrac{E_0}{k}.$

Note that the field becomes $1/k$ times by insertion of dielectric.

CAPACITORS IN SERIES AND PARALLEL COMBINATION

Series Combinations :

When capacitors are connected in series, the magnitude of charge Q on each capacitor is same. The potential difference across C_1 and C_2 is different i.e., V_1 and V_2.

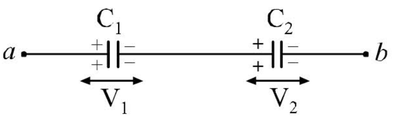
$Q = C_1 V_1 = C_2 V_2$

the total potential difference across combination is :

$V = V_1 + V_2$

$V = \dfrac{Q}{C_1} + \dfrac{Q}{C_2}$

$\dfrac{V}{Q} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$



The ratio Q/V is called as *the equivalent capacitance C* between point a and b.

The equivalent capacitance C is given by : $\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$

The potential difference across C_1 and C_2 is V_1 and V_2 respectively, given as follows :

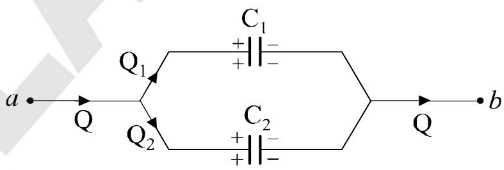
$$V_1 = \frac{C_2}{C_1 + C_2}; \qquad V_2 = \frac{C_1}{C_1 + C_2}V$$

In case of more than two capacitors, the relation is :

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} + \ldots\ldots\ldots$$

Parallel Combinations :

When capacitors are connected in parallel, the potential difference V across each is same and the charge on C_1, C_2 is different i.e., Q_1 and Q_2.



The total charge is Q given as :

$$Q = Q_1 + Q_2$$

$$Q = C_1V + C_2V$$

$$\frac{Q}{V} = C_1 + C_2$$

Equivalent capacitance between a and b is :

$$C = C_1 + C_2$$

The charges on capacitors is given as :

$$Q_1 = \frac{C_1}{C_1 + C_2}Q$$

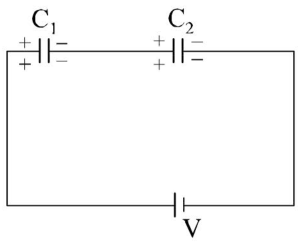
$$Q_2 = \frac{C_2}{C_1 + C_2}Q$$

In case of more than two capacitors, $C = C_1 + C_2 + C_3 + C_4 + C_5 + \ .\ .\ .\ .\ .\ .\ .$

Illustration - 5 *Two capacitors of capacitances $C_1 = 6\ \mu F$ and $C_2 = 3\ \mu F$ are connected in series across a cell of emf 18 V. Calculate :*

(a) *the equivalent capacitance*
(b) *the potential difference across each capacitor*
(c) *the charge on each capacitor.*

SOLUTION :



(a) $$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\Rightarrow \quad C = \frac{C_1 C_2}{C_1 + C_2} = \frac{6 \times 3}{6 + 3} = 2\ \mu F.$$

(b) $$V_1 = \frac{C_2}{C_1 + C_2} V = \frac{3}{6 + 3} \times 18 = 6 \text{ volts}$$

$$V_2 = \frac{C_1}{C_1 + C_2} V = \frac{6}{6 + 3} \times 18 = 12 \text{ volts}$$

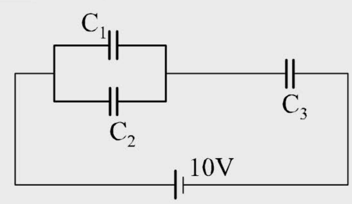
Note that the smaller capacitor C_2 has a larger potential difference across it.

(c) $Q_1 = Q_2 = C_1 V_1 = C_2 V_2 = CV$

charge on each capacitor $= CV$

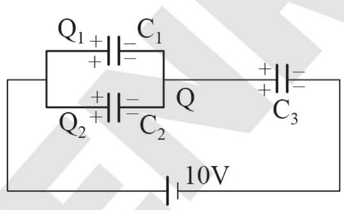
$= 2\ \mu F \times 18 \text{ volts} = 36\ \mu C$

Illustration - 6 *In the circuit, the capacitors are $C_1 = 15\ \mu F$, $C_2 = 10\ \mu F$, $C_3 = 25\ \mu F$.*



(a) *Find the equivalent capacitance of the circuit,*
(b) *Find the charge on each capacitor, and*
(c) *Find the potential difference across each capacitor.*

SOLUTION :



(a) $$C = \frac{(C_1 + C_2)\, C_3}{(C_1 + C_2) + C_3} = \frac{25 \times 25}{25 + 25}\ \mu F = 12.5\ \mu F$$

(b) Q = total charge supplied by the cell $= C\,V$

$Q = (12.5 \times 10)\ \mu C = 125\ \mu C$

charge on $C_1 = Q_1 = \dfrac{C_1}{C_1 + C_2} Q = 75\ \mu C$

charge on $C_2 = Q_2 = \dfrac{C_1}{C_1 + C_2} Q = 50\ \mu C$

charge on $C_3 = Q = 125\ \mu C$

(c) p.d across $C_1 = V_1$

$$= \frac{Q_1}{C_1} = \frac{75}{15} \text{ volts} = 5 \text{ volts.}$$

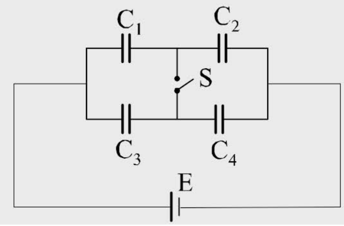
p.d.across $C_2 = V_2 = V_1 = 5$ volts.

p.d across $C_3 = Q_3 = \dfrac{Q_3}{C_3} = \dfrac{125}{25}$ volts $= 5$ volts .

Illustration - 7 *The emf of the cell in the circuit is 12 volts and the capacitors are : $C_1 = 1\ \mu F$, $C_2 = 3\ \mu F$, $C_3 = 2\ \mu F$, $C_4 = 4\ \mu F$. Calculate the charge on each capacitor and the total charge drawn from the cell when*

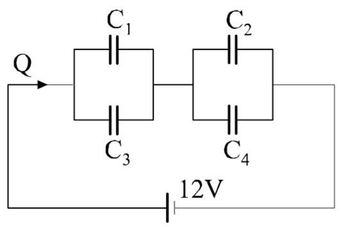
(a) *the switch S is closed*

(b) *the switch S is open.*



SOLUTION :

(a) Switch *S* is closed :



$$C = \frac{(C_1 + C_2)(C_2 + C_4)}{(C_1 + C_3) + (C_2 + C_4)}$$

$$\Rightarrow \quad C = \frac{3 \times 7}{3 + 7} = 2.1\ \mu F$$

total charge drawn from the cell is :

$$Q = C\,V = 2.1\ \mu F \times 12 \text{ volts} = 25.2\ \mu C$$

C_1, C_3 are in parallel and C_2, C_4 are in parallel.

Charge on C_1

$$Q_1 = \frac{C_1}{C_1 + C_3} Q = \frac{1}{1 + 2} \times 25.2\ C$$
$$= 8.4\ \mu C.$$

Charge on C_3

$$Q_3 = \frac{C_3}{C_1 + C_3} Q = \frac{2}{1 + 2} \times 25.2\ \text{C}$$
$$= 16.8 \mu\ \text{C}$$

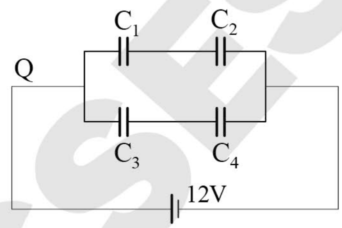
Charge on C_2

$$Q_2 = \frac{C_2}{C_2 + C_4} Q = \frac{3}{3 + 4} \times 25.2\ C$$
$$= 10.8\ \mu C$$

Charge on C_4

$$Q_4 = \frac{C_4}{C_2 + C_4} Q = \frac{4}{3 + 4} \times 25.2\ C$$
$$= 14.4\ \mu C$$

(b) Switch S is open :



$$C = \frac{C_1 C_2}{C_1 + C_2} + \frac{C_3\ C_4}{C_3 + C_4}$$

$$C = \frac{1 \times 3}{1 + 3} + \frac{2 \times 4}{2 + 4} = \frac{25}{12}\ \mu F$$

Total charge drawn from battery is :

$$Q = CV = \frac{25}{12} \times 12 = 25\ C.$$

C_1 and C_2 are in series and the potential difference across combination is 12 volts.

Charge on C_1 = charge on C_2

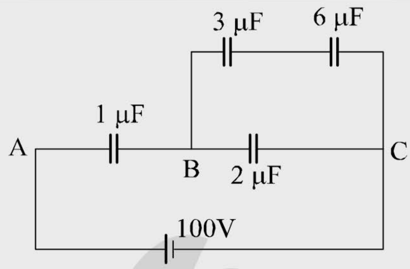
$$= \left(\frac{C_1 C_2}{C_1 + C_2}\right) V = \frac{3}{4} \times 12 = 9\ \mu C.$$

C_3 and C_4 are in series and the potential difference across combination is 12 volts.

charge on C_3 = charge on C_4

$$= \left(\frac{C_3 C_4}{C_3 + C_4}\right) V = \frac{8}{6} \times 12 = 16\ \mu C.$$

Illustration - 8 *In the circuit shown, the capacitances are :*

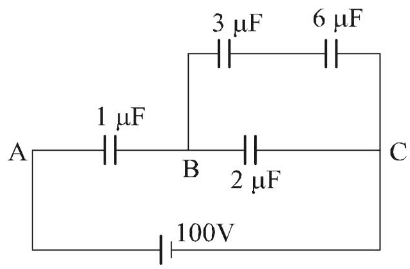


$C_1 = 1\ \mu F \qquad C_2 = 2\ \mu F$

$C_3 = 3\ \mu F \qquad C_4 = 6\ \mu F$

The emf of the cell is E = 100 volts. Find the charge and the potential difference across the capacitor C_4.

SOLUTION :



The capacitance between points B, C is :

$$C_{BC} = \frac{3 \times 6}{3+6} + 2 = 4\ \mu F$$

Potential difference across B, C is :

$$V_{BC} = \frac{C_1}{C_1 + C_{BC}} E = \frac{1}{1+4}(100)$$

= 20 volts.

Potential difference across C_4 is :

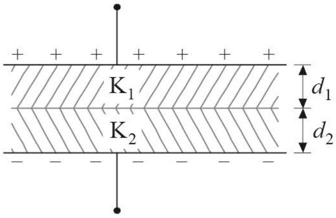
$$V_4 = \frac{3}{3+6} \times 20 = \frac{20}{3} \text{ volts}$$

Charge on capacitor $C_4 = C_4 V_4$

$$= 6 \times \frac{60}{9}\ \mu C = 40\ \mu C.$$

CAPACITOR WITH MORE THAN ONE DIELECTRIC SLABS

(I) A parallel plate capacitor contains two dielectric slabs of thickness d_1, d_2 and dielectric constants k_1 and k_2 respectively. The area of the capacitor plates and slabs is equal to A.



Considering the capacitor as a combination of two capacitors in series, the equivalent capacitance C is given by :

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C} = \frac{d_1}{k_1 \varepsilon_0 A} + \frac{d_2}{k_2 \varepsilon_0 A}$$

$$\Rightarrow \qquad C = \frac{\varepsilon_0 A}{\frac{d_1}{k_1} + \frac{d_2}{k_2}}$$

In general for more than one dielectric slab : $C = \dfrac{\varepsilon_0 A}{\sum \dfrac{d_i}{k_i}}$

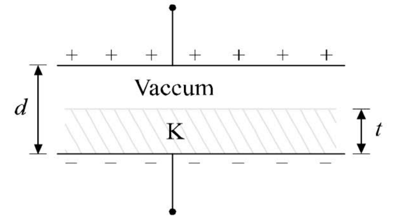
If V is the potential difference across the plates, the electric fields in the dielectrics are given as :

$$E_1 = \frac{V_1}{d_1} = \frac{Q}{C_1 d_1} = \frac{CV}{C_1 d_1} \qquad \left(use\ C_1 = \frac{k_1 \varepsilon_0 A}{d_1}\right)$$

$$E_1 = \frac{1}{\dfrac{d_1}{k_1} + \dfrac{d_2}{k_2}}\left(\frac{V}{k_1}\right) \qquad E_2 = \frac{1}{\dfrac{d_1}{k_1} + \dfrac{d_2}{k_2}}\left(\frac{V}{k_2}\right)$$

Note : $k_1 E_1 = k_2 E_2$ and $E_1 d_1 + E_2 d_2 = V$

(II) If there exits a dielectric slab of thickness t inside a capacitor whose plates are separated by distance d, the equivalent capacitance is given as :



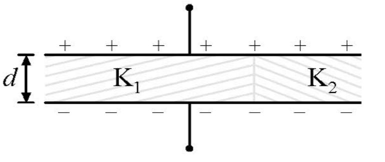
$$C = \frac{\varepsilon_0 A}{\dfrac{t}{k} + \dfrac{d-t}{1}} \qquad (k = 1 \text{ for vacuum})$$

$$C = \frac{\varepsilon_0 A}{\dfrac{t}{k} + d - t}$$

The equivalent capacitance is not affected by changing the distance of slab from the parallel plates.

If the slab is of metal, the equivalent capacitance is : $C = \dfrac{\varepsilon_0 A}{d - t}$ (for a metal, $k = \infty$)

(III) Consider a capacitor with two dielectric slabs of same thickness d placed inside it as shown. The slabs have dielectric constants k_1 and k_2 and areas A_1 and A_2 respectively. Treating the combination as two capacitors in parallel,



$C = C_1 + C_2$

$$C = \frac{k_1 \varepsilon_0 A_1}{d} + \frac{k_2 \varepsilon_0 A_2}{d} \quad \Rightarrow \quad C = \frac{\varepsilon_0}{d}[k_1 A_1 + k_2 A_2]$$

Illustration - 9 *A slab is inserted inside a parallel plate capacitor whose capacitance is 20 μF without the slab. The thickness of the slab is 0.6 times the separation between the plates. The capacitor is charged to a potential difference of 200 volts and then disconnected from the source. The slab was then removed from the gap. Find the work done in removing the slab if it is made of*

(a) *glass* ($k = 6$) **(b)** *metal.*

SOLUTION :

Let C_0, C be the capacitances before and after the insertion.

(a) With glass slab :

$$C_0 = \frac{\varepsilon_0 A}{d} \quad \text{and} \quad C = \frac{\varepsilon_0 A}{d - t + \frac{t}{k}}$$

where $t = 0.6\,d$

$$\Rightarrow C = \frac{C_0 d}{d - t + \frac{t}{k}} = \frac{20 \times 10^{-6}}{1 - 0.6 + \frac{0.6}{6}} = 40\ \mu F$$

Q = charge on capacitor = CV

$= 40 \times 200 \times 10^{-6}\ V = 8 \times 10^3\ \mu C.$

After removing the slab, the capacitance again becomes C_0.

$$\text{Work done} = \frac{Q^2}{2C_0} - \frac{Q^2}{2C} \qquad (= \text{gain in P.E.})$$

$$= \frac{8 \times 8 \times 10^{-6}}{2}\left[\frac{10^6}{20} - \frac{10^6}{40}\right] = 0.8\ J.$$

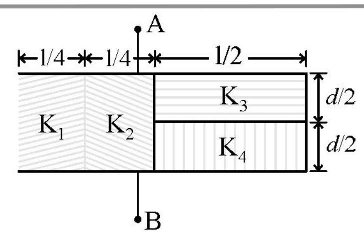
(b) with metal slab :

$$C = \frac{\varepsilon_0 A}{d - t} = \frac{C_0 d}{d - t} = 2.5\ C_0 = 50\ F.$$

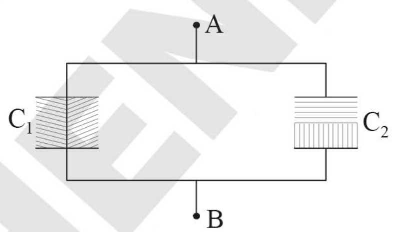
$$\text{Work done} = \frac{Q^2}{2}\left[\frac{1}{C_0} - \frac{1}{C}\right]$$

$$= \frac{\left(50 \times 10^{-6} \times 200\right)^2}{2}\left[\frac{10^6}{20} - \frac{10^6}{50}\right] = 1.5\ J$$

Illustration - 10 *The plates of the capacitor formed by inserting four dielectric slabs (as shown) have an area equal to S. Find the equivalent capacitance between A and B if* $2\,k_1 = 2\,k_2 = k_3 = k_4 = 5$.



SOLUTION :



Consider the capacitor as a parallel combination of C_1 and C_2.

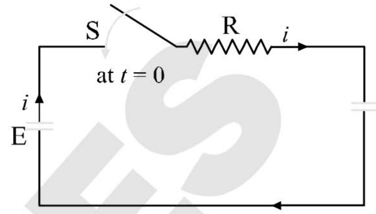
Net capacitance = $C_1 + C_2$

$$= \frac{\varepsilon_0}{d}\left(k_1 \frac{S}{4} + k_2 \frac{S}{4}\right) + \frac{\varepsilon_0\ S/2}{\frac{d/2}{k_3} + \frac{d/2}{k_4}}$$

$$= \frac{\varepsilon_0 S}{4d}\left(k_1 + k_2\right) + \frac{\varepsilon_0 S}{d}\left[\frac{k_3 k_4}{k_3 + k_4}\right]$$

CHARGING AND DISCHARGING OF A CAPACITOR

In the circuit considered so far, we have been concerned with the capacitors in the steady state i.e., the capacitors which have already been charged to their steady state voltages.



Now consider a circuit where an uncharged capacitor C is connected to a cell of emf E through a resistance R and a switch S as shown. At $t = 0$, the switch S is closed. The positive charge begins to flow from positive terminal of cell towards the upper capacitor plate and from lower plate to the negative terminal of the cell. Thus the upper plate of capacitor begins to acquire positive charge and lower plate becomes negatively charged. The voltage across capacitor begins to grow.

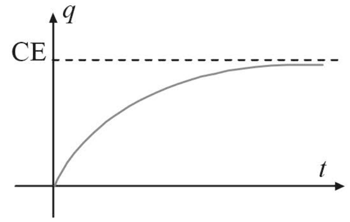
Let q, V_c be the charge and voltage on the capacitor at time t and i be the current.

Kirchoff's Law gives :

$$E - iR - V_c = 0$$

$$E - R\frac{dq}{dt} - \frac{q}{C} = 0 \qquad \left(\because \quad i = \frac{dq}{dt}\right)$$

$$\Rightarrow \qquad \frac{dq}{CE - q} = \frac{dt}{RC} \qquad \Rightarrow \qquad \int_0^q \frac{-dq}{CE - q} = \int_0^t \frac{-dt}{RC}$$

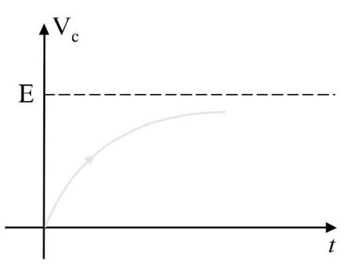


$$q(t) = CE\,(1 - e^{-t/RC})$$

This equation gives the expression for charge on capacitor as a function of time. The charge grows on the plate exponentially as shown on the graph. Note the following points.

1. In steady state : $t \to \infty$ and $q \to C\,E$
2. The voltage across capacitor also grows exponentially towards E.

$$V = \frac{q}{C} = E\,(1 - e^{-t/RC})$$



3. The time constant (τ) of the circuit is defined as the time after which the charge has grown upto $(1 - 1/e) = 0.63 \equiv 63\,\%$ of its steady-state value.

$$\tau = R\,C$$

4. From conservation of energy, we can see that :

$$\begin{matrix}\text{Energy supplied} \\ \text{by cell per sec}\end{matrix} = \begin{matrix}\text{Energy stored} \\ \text{capacitor}\end{matrix} + \begin{matrix}\text{Heat disipated} \\ \text{in R per sec}\end{matrix}$$

$$Ei = iV_c + i^2 R$$

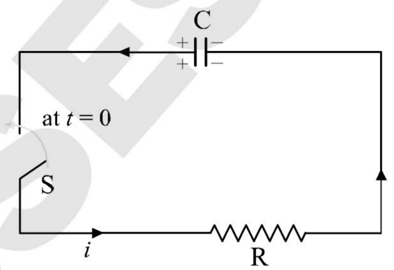
$$E = V_c + iR$$

$$E = \frac{q}{C} + R\frac{dq}{dt}$$

(Note that $i = \frac{dq}{dt}$, as q is increasing and hence $\frac{dq}{dt}$ is positive)

Discharging of a Capacitor :

If we connect a charged capacitor C across a resistance R, the capacitor begins to discharge through R. The excess positive charge on high potential plate flows through R to the negative plate and in steady state, the capacitor plates become uncharged. As the charge on plates decreases with time, dq/dt is negative and hence :

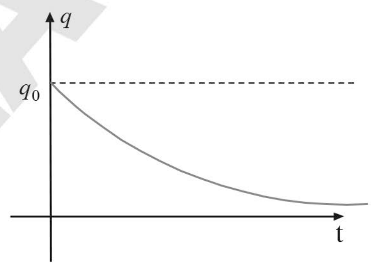


$$i = -\frac{dq}{dt}$$

From Kirchoff's Law :

$$V_c = i\,R$$

$$\Rightarrow \quad \frac{q}{C} = -R\frac{dq}{dt} \quad \Rightarrow \quad \int_{q_0}^{q} \frac{dq}{q} = \int_0^t -\frac{dt}{RC}$$



where q_0 is the charge an capacitor at $t = 0$.

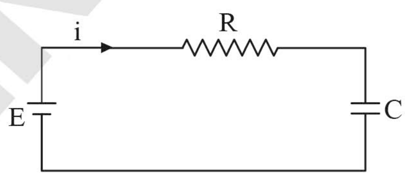
$$q(t) = q_0\, e^{-t/RC}$$

Illustration - 11 *A 3×10^6 ohm resistor and a 1 μF capacitor are connected in a single-loop circuit with a seat of emf with E = 4 volts. At 1 sec after the connection is made, what are the rates at which :*

(a) *the charge of the capacitor is increasing,*
(b) *energy is being stored in the capacitor,*
(c) *joule heat is appearing in the resistor,*
(d) *energy is being delivered by the seat of emf ?*

SOLUTION :

$E = 4\ V$
$R = 3 \times 10^6$ ohm
$C = 1\ \mu F$



(a) $$i = \frac{dq}{dt} = \frac{d}{dt}\left[CE\left(1 - e^{1t/RC}\right)\right]$$

$$= \frac{CE}{RC} e^{-t/RC} = \frac{E}{R} e^{-t/RC}$$

At $t = 1s$:

$$\frac{dq}{dt} = \frac{4}{3 \times 10^6} e^{-1/RC}$$

$$= \frac{4}{3} \times 10^{-6} e^{-1/3} = 9.6 \times 10^{-7}\ C/s$$

(b) $$\frac{dU}{dt} = iV_c = \frac{iq}{C}$$

$$= \frac{1}{C}\frac{E}{R} e^{-t/RC} \; CE\left(1-e^{-t/RC}\right)$$

$$\frac{dU}{dt} = \frac{E^2}{R} e^{-1/3}\left(1-e^{-1/3}\right)$$

$$= \frac{16}{3\times 10^6} e^{-1/3}\left(1-e^{-1/3}\right) = 1.1\times 10^{-6}\ W$$

(c) $P = i^2 R$

$= (9.6 \times 10^{-7})^2 \times 3 \times 10^6$

$= 2.7 \times 10^{-6}\ W.$

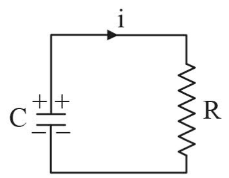
(d) Energy supplied/s $= E\,i$

$= 4\,(9.6 \times 10^{-7}) = 3.8 \times 10^{-6}\ W.$

Illustration - 12 *A 0.05 μF capacitor is charged to a potential of 200 V and is then permitted to discharge through 10 $M\,\Omega$ resistor. How much time is required for the charge to decrease to :*

(a) $1/e$ (b) $1/e^2$ *of its initial value ?*

SOLUTION :



At $t = 0$, charge on capacitor is q_0.

$q_0 = 0.05 \times 10^{-6} \times 200 = 1 \times 10^{-5}\ C.$

$q = q_0\, e^{-t/RC}$

(a) $q = \frac{1}{e} q_0 \;\Rightarrow\; \frac{q_0}{e} = q_0 e^{-t/RC}$

$\Rightarrow \quad e^{-1} = e^{-t/RC}$

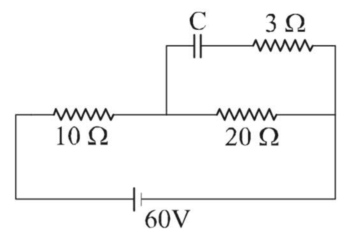
$\Rightarrow \quad t = RC = 10 \times 10^6 \times 0.05 \times 10^{-6}\ s$

$= 0.5\ s.$

(b) $q = \frac{1}{e^2} q_0 \;\Rightarrow\; \frac{q_0}{e^2} = q_0 e^{-t/RC}$

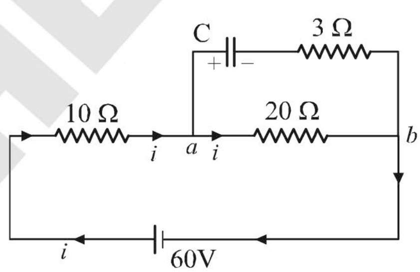
$\Rightarrow \quad t = 2RC = 1s.$

Illustration - 13 *The circuit shown on the right is in steady state. Find the charge on the capacitor plates and the energy stored in the capacitor $C = 4\ \mu F$.*



SOLUTION :

When the circuit is in steady state, there is no current through the capacitor and hence there is no current through the 3 ohm resistor.



All the current supplied by battery goes through the 10 ohm resistor and 20 ohm resistor which appear in series.

$$\Rightarrow \quad i = \frac{60}{10+20} = 2A$$

$\Rightarrow$ potential difference across capacitor plates

$= 7$ p.d. across $ab = i \times 20$

$= 2 \times 20 = 40$ volts.

Charge on capacitor $= CV_{ab} = 4 \times 40\ \mu C$

$= 160\ \mu C.$

Energy stored $= \frac{1}{2} CV_{ab}^2$

$= \frac{1}{2}(4\mu F) \times (40\ V)^2 = 3200\ \mu J$